THE BIG BOOK OF GAMES / PUZZLE BOOK

The Great Solo Challenge

Puzzles for the most determined minds.

40 CHALLENGES / EXPERT AND EXTREME

Sudoku, numbers, spatial puzzles and strategy

Created by Tokiniaina Hervé

Sudoku and variants

Click a page number to jump to the puzzle or solution. The solutions follow the complete set of puzzles.

Challenge	Puzzle	Solution
SOLO-001 The Ninth Lock Classic Sudoku / EXPERT	7	50
SOLO-002 The Hidden Crossroads Classic Sudoku / EXTREME	8	52
SOLO-003 The Inverted Mesh Classic Sudoku / EXPERT	9	54
SOLO-004 Branches of Silence Classic Sudoku / EXTREME	10	56
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Numbers and deduction

Click a page number to jump to the puzzle or solution. The solutions follow the complete set of puzzles.

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Click a page number to jump to the puzzle or solution. The solutions follow the complete set of puzzles.

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PART ONE

The Challenges

All you need is a pencil. The solutions are in Part Two.

The Ninth Lock

Classic Sudoku / EXPERT

Complete the grid completely. Only one solution respects all constraints.

Materials. Pencil and gum; no commercial material.

Rules

- Fill in the 81 boxes with numbers from 1 to 9. Each row and column contains each of the nine digits exactly once.

- The coordinates L1C1 to L9C9 are read from the top left corner. No calculation is made on the coordinates numbers.

- Each of the nine blocks of 3 × 3 also contains numbers 1 to 9 without repetition.

Puzzle

The empty boxes are to be completed. The given numbers are fixed. The answer and proof are on another page.

	1	2	3	4	5	6	7	8	9
1				7			6		
2			5						7
3		8			2	4			
4				5				9	
5	1	3				9			
6		4					3	6	
7	7			2					1
8	3				5		7		
9				3	7		4		

Working space

Solution p. 50

The Hidden Crossroads

Classic Sudoku / EXTREME

Complete the grid completely. Only one solution respects all constraints.

Materials. Pencil and gum; no commercial material.

Rules

- Fill in the 81 boxes with numbers from 1 to 9. Each row and column contains each of the nine digits exactly once.

- The coordinates L1C1 to L9C9 are read from the top left corner. No calculation is made on the coordinates numbers.

- Each of the nine blocks of 3 × 3 also contains numbers 1 to 9 without repetition.

Puzzle

The empty boxes are to be completed. The given numbers are fixed. The answer and proof are on another page.

	1	2	3	4	5	6	7	8	9
1		4				3			9
2				9					6
3								5	
4	2		9			4			
5		1						7	
6			7	8	2				4
7	3				8		2	1	
8					3			8	
9		5				1			

Working space

Solution p. 52

The Inverted Mesh

Classic Sudoku / EXPERT

Complete the grid completely. Only one solution respects all constraints.

Materials. Pencil and gum; no commercial material.

Rules

- Fill in the 81 boxes with numbers from 1 to 9. Each row and column contains each of the nine digits exactly once.

- The coordinates L1C1 to L9C9 are read from the top left corner. No calculation is made on the coordinates numbers.

- Each of the nine blocks of 3 × 3 also contains numbers 1 to 9 without repetition.

Puzzle

The empty boxes are to be completed. The given numbers are fixed. The answer and proof are on another page.

	1	2	3	4	5	6	7	8	9
1					7		9	1	
2	7				3				
3				4	9				
4			9						8
5			3			4		5	
6		8		5		2		7	
7	2	1				7			5
8				8			2		
9			6				7		

Working space

Solution p. 54

Branches of Silence

Classic Sudoku / EXTREME

Complete the grid completely. Only one solution respects all constraints.

Materials. Pencil and gum; no commercial material.

Rules

- Fill in the 81 boxes with numbers from 1 to 9. Each row and column contains each of the nine digits exactly once.

- The coordinates L1C1 to L9C9 are read from the top left corner. No calculation is made on the coordinates numbers.

- Each of the nine blocks of 3 × 3 also contains numbers 1 to 9 without repetition.

Puzzle

The empty boxes are to be completed. The given numbers are fixed. The answer and proof are on another page.

	1	2	3	4	5	6	7	8	9
1	7		2		8			4	
2	3			1		2			
3				6	5				
4									
5	8	5			7				2
6							9	6	
7			8	4			5	3	
8						5	6		
9		3				1			

Working space

Solution p. 56

The Two Crossings

Diagonal Sudoku / EXPERT

Complete the grid completely. Only one solution respects all constraints.

Materials. Pencil and gum; no commercial material.

Rules

- Fill in the 81 boxes with numbers from 1 to 9. Each row and column contains each of the nine digits exactly once.

- The coordinates L1C1 to L9C9 are read from the top left corner. No calculation is made on the coordinates numbers.

- Each of the nine blocks of 3 × 3 also contains numbers 1 to 9 without repetition.

- The two large diagonals each contain numbers 1 to 9 without repetition. This constraint adds to the blocks.

Puzzle

The empty boxes are to be completed. The given numbers are fixed. The answer and proof are on another page.

	1	2	3	4	5	6	7	8	9
1					5				
2		9	3				7		
3		4	5						8
4		7		4			6		
5				1					
6									3
7				5	9			4	
8					4				
9			7					3	

Working space

Solution p. 58

The Stubborn Mirror

Diagonal Sudoku / EXTREME

Complete the grid completely. Only one solution respects all constraints.

Materials. Pencil and gum; no commercial material.

Rules

- Fill in the 81 boxes with numbers from 1 to 9. Each row and column contains each of the nine digits exactly once.

- The coordinates L1C1 to L9C9 are read from the top left corner. No calculation is made on the coordinates numbers.

- Each of the nine blocks of 3 × 3 also contains numbers 1 to 9 without repetition.

- The two large diagonals each contain numbers 1 to 9 without repetition. This constraint adds to the blocks.

Puzzle

The empty boxes are to be completed. The given numbers are fixed. The answer and proof are on another page.

	1	2	3	4	5	6	7	8	9
1						2		1	3
2	4			6			8		9
3					4				
4									
5						6			
6	7		1	4					
7	5				6	9	7		
8									
9				1	2	5			

Working space

Solution p. 60

The Living Border

Irregular Sudoku / EXPERT

Complete the grid completely. Only one solution respects all constraints.

Materials. Pencil and gum; no commercial material.

Rules

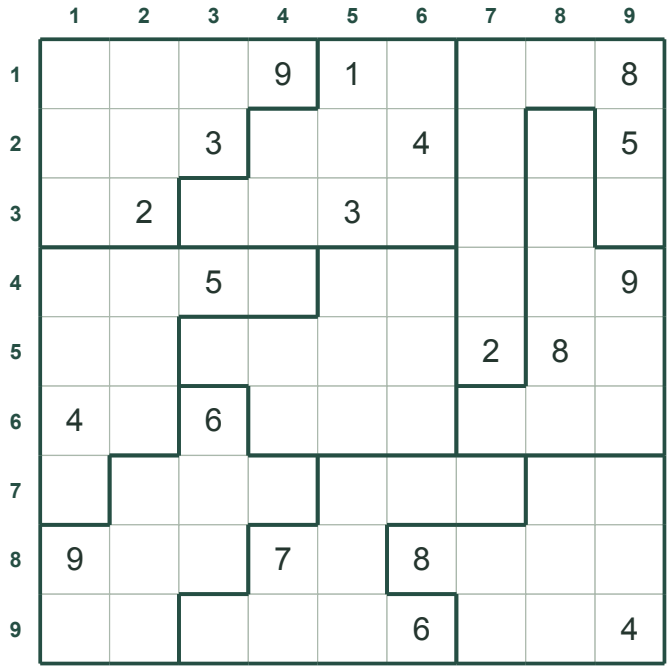
- Fill in the 81 boxes with numbers from 1 to 9. Each row and column contains each of the nine digits exactly once.

- The coordinates L1C1 to L9C9 are read from the top left corner. No calculation is made on the coordinates numbers.

- The nine regions delimited by a thick line also contain figures 1 to 9 without repetition.

Puzzle

The empty boxes are to be completed. The given numbers are fixed. The answer and proof are on another page.



Working space

Solution p. 62

The Nine Territories

Irregular Sudoku / EXTREME

Complete the grid completely. Only one solution respects all constraints.

Materials. Pencil and gum; no commercial material.

Rules

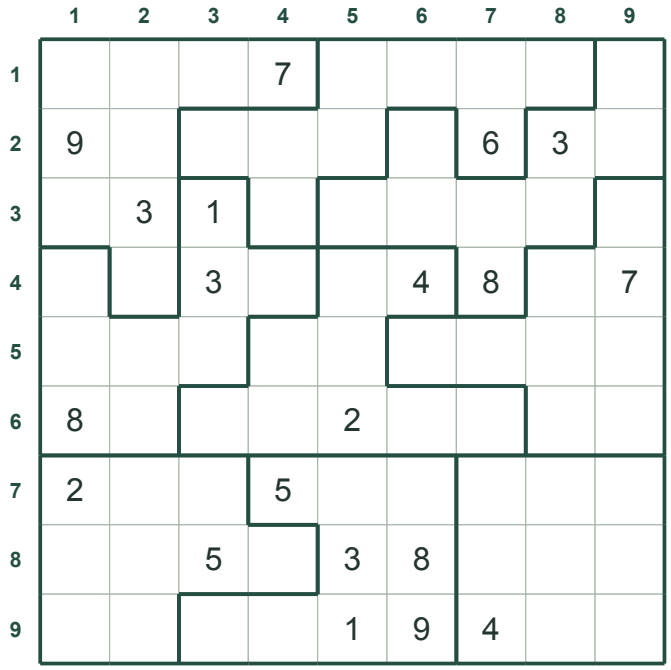
- Fill in the 81 boxes with numbers from 1 to 9. Each row and column contains each of the nine digits exactly once.

- The coordinates L1C1 to L9C9 are read from the top left corner. No calculation is made on the coordinates numbers.

- The nine regions delimited by a thick line also contain figures 1 to 9 without repetition.

Puzzle

The empty boxes are to be completed. The given numbers are fixed. The answer and proof are on another page.



Working space

Solution p. 64

Captive Sums

Killer Sudoku / EXPERT

Complete the grid completely. Only one solution respects all constraints.

Materials. Pencil and gum; no commercial material.

Rules

- Fill in the 81 boxes with numbers from 1 to 9. Each row and column contains each of the nine digits exactly once.

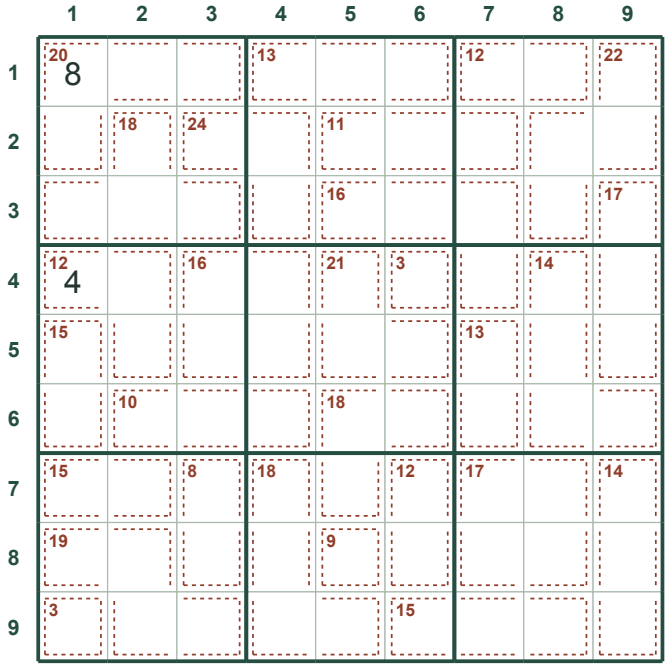
- The coordinates L1C1 to L9C9 are read from the top left corner. No calculation is made on the coordinates numbers.

- Each of the nine blocks of 3 × 3 also contains numbers 1 to 9 without repetition.

- Each dotted cage carries a total. The numbers of the cage have this sum and are all different; this prohibition also applies between two cells of the cage that do not share a line, column or block. The few pre-filled figures are part of the problem.

Puzzle

The empty boxes are to be completed. The given numbers are fixed. The answer and proof are on another page.



Solid lines mark regions. Dashed lines and small numbers mark cages and their totals.

Working space

Solution p. 66

Unyielding Totals

Killer Sudoku / EXTREME

Complete the grid completely. Only one solution respects all constraints.

Materials. Pencil and gum; no commercial material.

Rules

- Fill in the 81 boxes with numbers from 1 to 9. Each row and column contains each of the nine digits exactly once.

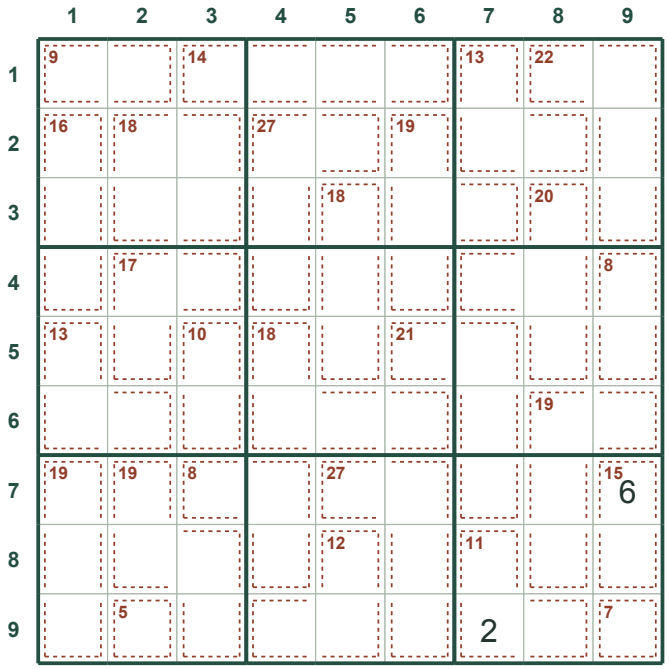
- The coordinates L1C1 to L9C9 are read from the top left corner. No calculation is made on the coordinates numbers.

- Each of the nine blocks of 3 × 3 also contains numbers 1 to 9 without repetition.

- Each dotted cage carries a total. The numbers of the cage have this sum and are all different; this prohibition also applies between two cells of the cage that do not share a line, column or block. The few pre-filled figures are part of the problem.

Puzzle

The empty boxes are to be completed. The given numbers are fixed. The answer and proof are on another page.



Solid lines mark regions. Dashed lines and small numbers mark cages and their totals.

Working space

Solution p. 68

The Eight Inks

Advanced Mastermind / EXPERT

Find the only ordered code of 5 digits.

Materials. Pencil, gum and gridded paper; no commercial game required.

Rules

- Each position contains a number from 0 to 7. Repeats are allowed. The code has exactly 5 positions; an initial zero is allowed.

- "Good place" includes the positions whose number and place are correct. "Unplaced" counts the other numbers present, respecting the multiplicity.

- To count the wrong placed, first matches the well placed numbers, then each remaining occurrence at the most once. The indices do not give the location of the incorrectly placed numbers.

- All indices are simultaneously true. There is no hidden information about colors, parity or missing numbers.

Puzzle

The table reads from top to bottom. The proposals are tests and are not already confirmed number positions.

Write down the possible multiplicities of each number, then look for arrangements compatible with all scores.

Test	Guess	Exact matches	Misplaced matches
1	02553	0	2
2	62532	1	0
3	05062	0	2
4	73375	1	2

Working space

Solution p. 70

The Silent Lock

Advanced Mastermind / EXPERT

Find the only ordered code of 5 digits.

Materials. Pencil, gum and gridded paper; no commercial game required.

Rules

- Each position contains a number from 0 to 7. Repeats are allowed. The code has exactly 5 positions; an initial zero is allowed.

- "Good place" includes the positions whose number and place are correct. "Unplaced" counts the other numbers present, respecting the multiplicity.

- To count the wrong placed, first matches the well placed numbers, then each remaining occurrence at the most once. The indices do not give the location of the incorrectly placed numbers.

- All indices are simultaneously true. There is no hidden information about colors, parity or missing numbers.

Puzzle

The table reads from top to bottom. The proposals are tests and are not already confirmed number positions.

Write down the possible multiplicities of each number, then look for arrangements compatible with all scores.

Test	Guess	Exact matches	Misplaced matches
1	22543	1	3
2	41050	1	0
3	70302	1	2
4	37433	1	1

Working space

Solution p. 72

Six Slots Without an Echo

Advanced Mastermind / EXTREME

Find the only orderly code of 6 digits.

Materials. Pencil, gum and gridded paper; no commercial game required.

Rules

- Each position contains a number from 0 to 7. Repeats are allowed. The code has exactly 6 positions; an initial zero is allowed.

- "Good place" includes the positions whose number and place are correct. "Unplaced" counts the other numbers present, respecting the multiplicity.

- To count the wrong placed, first matches the well placed numbers, then each remaining occurrence at the most once. The indices do not give the location of the incorrectly placed numbers.

- All indices are simultaneously true. There is no hidden information about colors, parity or missing numbers.

Puzzle

The table reads from top to bottom. The proposals are tests and are not already confirmed number positions.

Write down the possible multiplicities of each number, then look for arrangements compatible with all scores.

Test	Guess	Exact matches	Misplaced matches
1	345125	1	1
2	073421	0	4
3	573237	2	1
4	531562	2	0

Working space

Solution p. 74

The Chamber of Duplicates

Advanced Mastermind / EXTREME

Find the only orderly code of 6 digits.

Materials. Pencil, gum and gridded paper; no commercial game required.

Rules

- Each position contains a number from 0 to 7. Repeats are allowed. The code has exactly 6 positions; an initial zero is allowed.

- "Good place" includes the positions whose number and place are correct. "Unplaced" counts the other numbers present, respecting the multiplicity.

- To count the wrong placed, first matches the well placed numbers, then each remaining occurrence at the most once. The indices do not give the location of the incorrectly placed numbers.

- All indices are simultaneously true. There is no hidden information about colors, parity or missing numbers.

Puzzle

The table reads from top to bottom. The proposals are tests and are not already confirmed number positions.

Write down the possible multiplicities of each number, then look for arrangements compatible with all scores.

Test	Guess	Exact matches	Misplaced matches
1	474563	1	2
2	013237	1	3
3	163003	2	3
4	635050	0	3
5	042546	1	2

Working space

Solution p. 77

The Twin Carries

Cryptarithm / EXPERT

Determine the value of each of the ten letters, then write the two additions in digits.

Materials. Pencil, gum and gridded paper; no commercial game required.

Rules

- Each letter A to J represents a number from 0 to 9. Two different letters always represent different numbers.
- The ten digits are therefore used exactly once in the substitution; they may reappear several times in the numbers by repetition of the letters.
- No number starts with zero. Both additions use exactly the same substitution. The notation is ordinary decimal.

Puzzle

The two additions are simultaneously true. The letter names do not bear any value index.

Addition	First term	Second term	Result
1	DDFB	JHEG	AHIBD
2	EFIF	JEEC	ADBFE

Working space

Solution p. 79

The Second Addition

Cryptarithm / EXPERT

Determine the value of each of the ten letters, then write the two additions in digits.

Materials. Pencil, gum and gridded paper; no commercial game required.

Rules

- Each letter A to J represents a number from 0 to 9. Two different letters always represent different numbers.
- The ten digits are therefore used exactly once in the substitution; they may reappear several times in the numbers by repetition of the letters.
- No number starts with zero. Both additions use exactly the same substitution. The notation is ordinary decimal.

Puzzle

The two additions are simultaneously true. The letter names do not bear any value index.

Addition	First term	Second term	Result
1	DBBG	GGJE	CGDF
2	GCIF	FHGC	DIIAG

Working space

Solution p. 81

Ten Letters, Two Locks

Cryptarithm / EXTREME

Determine the value of each of the ten letters, then write the two additions in digits.

Materials. Pencil, gum and gridded paper; no commercial game required.

Rules

- Each letter A to J represents a number from 0 to 9. Two different letters always represent different numbers.
- The ten digits are therefore used exactly once in the substitution; they may reappear several times in the numbers by repetition of the letters.
- No number starts with zero. Both additions use exactly the same substitution. The notation is ordinary decimal.

Puzzle

The two additions are simultaneously true. The letter names do not bear any value index.

Addition	First term	Second term	Result
1	JIIB	FHBF	IDIF
2	CGJG	EAII	IDHE

Working space

Solution p. 83

The Seven Staircases

Futoshiki / EXPERT

Complete square 7 × 7 with figures from 1 to 7.

Materials. Pencil, gum and gridded paper; no commercial game required.

Rules

- Each row and column contains exactly once every 1-7 digit. There are no 3 × 3 regions and no diagonal constraints.

- The printed values are fixed. All inequalities in the list are strict and must be respected simultaneously.

- LrCc is the line r and column c, counted from the top and left. A box marked " ·" is to be filled in.

Puzzle

All useful inequalities are given below; the neighbourhoods not mentioned do not bear any additional constraints. L5C1 < L4C1 L6C6 < L5C6 L3C6 < L3C7 L2C6 < L2C7 L6C2 < L7C2 L4C1 < L3C1 L7C7 < L7C6 L5C7 < L6C7 L5C2 < L4C2 L1C4 < L1C5 L1C4 < L2C4 L6C1 < L7C1 L6C4 < L7C4 L6C6 < L6C5 L4C2 < L3C2 L4C5 < L3C5 L2C7 < L3C7

	1	2	3	4	5	6	7
1	·	·	·	2	·	5	·
2	6	·	·	·	4	·	·
3	·	·	3	·	5	·	·
4	·	·	·	6	·	·	·
5	·	·	·	·	·	·	·
6	·	·	·	·	·	·	7
7	·	·	5	·	·	6	3

Working space

Solution p. 85

The Eight Passages

Futoshiki / EXTREME

Complete square 8 × 8 with figures from 1 to 8.

Materials. Pencil, gum and gridded paper; no commercial game required.

Rules

- Each row and column contains exactly once every 1 to 8 digit. There are no 3 × 3 regions and no diagonal constraints.

- The printed values are fixed. All inequalities in the list are strict and must be respected simultaneously.

- LrCc is the line r and column c, counted from the top and left. A box marked " ·" is to be filled in.

Puzzle

All useful inequalities are given below; the neighbourhoods not mentioned do not bear any additional constraints. L4C1 < L4C2 L5C6 < L5C7 L8C2 < L7C2 L1C5 < L1C4 L6C2 < L6C3 L1C2 < L2C2 L5C1 < L4C1 L4C5 < L3C5 L2C3 < L2C2 L4C3 < L4C2 L6C1 < L6C2 L3C6 < L3C7 L7C3 < L6C3 L6C4 < L5C4 L8C8 < L8C7 L5C2 < L4C2 L4C2 < L3C2 L1C2 < L1C3 L6C1 < L5C1

	1	2	3	4	5	6	7	8
1	·	5	·	·	2	7	·	·
2	·	·	2	3	·	·	·	·
3	6	·	·	·	·	·	·	·
4	·	·	·	·	·	·	·	·
5	·	·	4	·	·	·	8	·
6	·	·	·	·	·	8	·	1
7	·	·	·	5	8	2	·	·
8	·	·	·	6	3	·	·	·

Working space

Solution p. 89

Columns Under Tension

Futoshiki / EXTREME

Complete square 8 × 8 with figures from 1 to 8.

Materials. Pencil, gum and gridded paper; no commercial game required.

Rules

- Each row and column contains exactly once every 1 to 8 digit. There are no 3 × 3 regions and no diagonal constraints.

- The printed values are fixed. All inequalities in the list are strict and must be respected simultaneously.

- LrCc is the line r and column c, counted from the top and left. A box marked " ·" is to be filled in.

Puzzle

All useful inequalities are given below; the neighbourhoods not mentioned do not bear any additional constraints. L4C5 < L4C4 L8C6 < L7C6 L1C4 < L1C3 L2C3 < L1C3 L2C2 < L2C3 L3C6 < L3C7 L1C7 < L2C7 L6C6 < L6C7 L6C6 < L5C6 L8C1 < L8C2 L8C3 < L7C3 L7C4 < L7C5 L3C1 < L2C1 L6C2 < L5C2 L6C4 < L6C5 L5C6 < L4C6 L6C2 < L7C2 L7C1 < L8C1 L7C5 < L7C6

	1	2	3	4	5	6	7	8
1	·	·	·	·	·	2	·	·
2	·	·	·	5	·	·	·	·
3	·	·	·	4	·	·	5	·
4	·	5	·	·	·	·	8	4
5	·	4	3	6	2	·	·	·
6	·	·	·	·	6	3	·	·
7	·	·	·	·	·	·	·	6
8	·	·	·	·	1	·	·	5

Working space

Solution p. 93

Crossing Intervals

Nonogram / EXPERT

Black out the boxes of this grid 12 × 12 to satisfy all series.

Materials. Pencil and gum; grid below.

Rules

- A series shows the lengths of the consecutive black blocks in the order, from left to right for a line and from top to bottom for a column.

- Two adjacent blocks are separated by at least one empty box. Non-blacked boxes must remain empty. "0" indicates a completely empty line.

- The LrCc coordinates refer to line r, column c, counted from the upper left corner.

Puzzle

The series and possible pre-filled boxes are the only data of the problem. ■ = already black box, · = already proven empty box; the white boxes without symbol remain unknown. No symmetry or recognizable image should be assumed.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
	1 3 3	1 2 1 2	1 2 1 1 1	2 2 3	3 3	1 3 1 2	1 1 1 3	1 2 1	1 1 1 1 1	1 1 1 1	1 3 1	1 2 1 1 1
L1 1 5 1												
L2 1 2 1 2												
L3 1 1 1												
L4 3 1 1 3												
L5 3 2 1 2												
L6 1 1 1												
L7 3 1 1 1												
L8 1 3												
L9 1 5												
L10 1 2 1 1 1												
L11 2 1 1 2												
L12 2 1 2 1												

Working space

Solution p. 97

The Suspended Weave

Nonogram / EXPERT

Black out the boxes of this grid 12 × 12 to satisfy all series.

Materials. Pencil and gum; grid below.

Rules

- A series shows the lengths of the consecutive black blocks in the order, from left to right for a line and from top to bottom for a column.

- Two adjacent blocks are separated by at least one empty box. Non-blacked boxes must remain empty. "0" indicates a completely empty line.

- The LrCc coordinates refer to line r, column c, counted from the upper left corner.

Puzzle

The series and possible pre-filled boxes are the only data of the problem. ■ = already black box, · = already proven empty box; the white boxes without symbol remain unknown. No symmetry or recognizable image should be assumed.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12
	1 5 2	1 7 2	2 4 2	1 1 3 1	1 1 1 1	2 1 1	1 2 1 3	2 2	2 2	2 1 1 1 1	1 3	4 2
L1 2 2 1 1												
L2 1 1 1 1												
L3 4 1 1												
L4 2 3 3												
L5 2 2 2												
L6 3 1 2												
L7 4 1 1												
L8 4 1 1												
L9 4 1 2												
L10 1 2												
L11 5 2 1												
L12 3 1 1												

Working space

Solution p. 101

The Fragmented Grid

Nonogram / EXTREME

Black out the boxes of this 16 × 16 grid to satisfy all series.

Materials. Pencil and gum; grid below.

Rules

- A series shows the lengths of the consecutive black blocks in the order, from left to right for a line and from top to bottom for a column.

- Two adjacent blocks are separated by at least one empty box. Non-blacked boxes must remain empty. "0" indicates a completely empty line.

- The LrCc coordinates refer to line r, column c, counted from the upper left corner.

Puzzle

The series and possible pre-filled boxes are the only data of the problem. ■ = already black box, · = already proven empty box; the white boxes without symbol remain unknown. No symmetry or recognizable image should be assumed.

C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16
1	1	2	1	2	1	2	1	1	1	1	1	2	1	4	4
1	1	1	1	2	1	1	2	1	1	6	7	1	1	2	1
1	1	1		2	1	1	1	2	1	2	3	1	1	1	1
1	1	1		1	2	2		2	3			3	6	3	1
2	1	1		1	1			2	1			1		1	
	1														
	1														
	1														

L1	1 1 2															
L2	1 2 2 1 1 1															
L3	2 1 2															
L4	1 1 2 2 2										·					
L5	2 1 1 1 2															
L6	1 1 3 1															
L7	3 2 3 3															
L8	2 2 1															
L9	1 3 1 4															
L10	1 1 3 1 2		■													
L11	1 1 1 2															
L12	1 1 3 2															
L13	1 2 2 1 3			·												
L14	1 1 1 1 1 2															
L15	1 2 2 1 1															
L16	3 1 3 3															

Working space

Solution p. 105

The Hidden Border

Nonogram / EXTREME

Black out the boxes of this 16 × 16 grid to satisfy all series.

Materials. Pencil and gum; grid below.

Rules

- A series shows the lengths of the consecutive black blocks in the order, from left to right for a line and from top to bottom for a column.

- Two adjacent blocks are separated by at least one empty box. Non-blacked boxes must remain empty. "0" indicates a completely empty line.

- The LrCc coordinates refer to line r, column c, counted from the upper left corner.

Puzzle

The series and possible pre-filled boxes are the only data of the problem. ■ = already black box, · = already proven empty box; the white boxes without symbol remain unknown. No symmetry or recognizable image should be assumed.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16
	1 1 4	5 1 2	4 1 3	2 4 2	2 5 1	1 3 1 1 1	4 1 1	2 1 2 1 1	4 3 1 1	1 9 1	4 1 1 1 1	2 1 1 1	2 2 5	1 2 1 2	4 1 1 1	1 2 2 1 1 1
L1 1 3 3 1 1																
L2 9 1																
L3 2 1 1 1 3																
L4 2 1 1 1 3 2																
L5 2 2 1 1 2																
L6 5 3 2											■					
L7 2 3 2 2 1																
L8 1 2 5 2												·				
L9 1 2 1 1 1																
L10 1 1 1 2 3																
L11 2 2 1									·							
L12 1 1 1 1 2																
L13 1 2 1 2 1																
L14 1 1 1 1 3																
L15 1 3																
L16 1 3 1																

Working space

Solution p. 111

The Silent Checkpoints

Hamiltonian path / EXPERT

Number all boxes from 1 to 36 to form a single orthogonal path without repetition.

Materials. Pencil, gum and grill; no connection required.

Rules

- The successive numbers occupy two boxes sharing one side; no diagonal displacement.
- Each box receives exactly a number of 1 to 36. Print marks remain in their place.
- The course starts in 1 and ends in 36; it does not have to go back to the start.

Puzzle

The white boxes are all usable. The boxes without clues are not obstacles.

	1	2	3	4	5	6
1	1					
2					8	
3						
4			28			
5	15	18			33	36
6						35

Working space

Solution p. 117

The One-Way Crossing

Hamiltonian path / EXPERT

Number all boxes from 1 to 36 to form a single orthogonal path without repetition.

Materials. Pencil, gum and grill; no connection required.

Rules

- The successive numbers occupy two boxes sharing one side; no diagonal displacement.
- Each box receives exactly a number of 1 to 36. Print marks remain in their place.
- The course starts in 1 and ends in 36; it does not have to go back to the start.

Puzzle

The white boxes are all usable. The boxes without clues are not obstacles.

	1	2	3	4	5	6
1						36
2	28					
3	21				1	
4						
5		12				4
6		9			6	

Working space

Solution p. 119

The Maze Without Walls

Hamiltonian path / EXTREME

Number all boxes from 1 to 36 to form a single orthogonal path without repetition.

Materials. Pencil, gum and grill; no connection required.

Rules

- The successive numbers occupy two boxes sharing one side; no diagonal displacement.
- Each box receives exactly a number of 1 to 36. Print marks remain in their place.
- The course starts in 1 and ends in 36; it does not have to go back to the start.

Puzzle

The white boxes are all usable. The boxes without clues are not obstacles.

	1	2	3	4	5	6
1						
2		23				
3						
4		1				
5		36			17	
6		33	32			

Working space

Solution p. 121

The Eight Shapes

Pentomino tiling / EXPERT

Pave the rectangle 5 × 8 completely with the 8 pentominos given, without overlap.

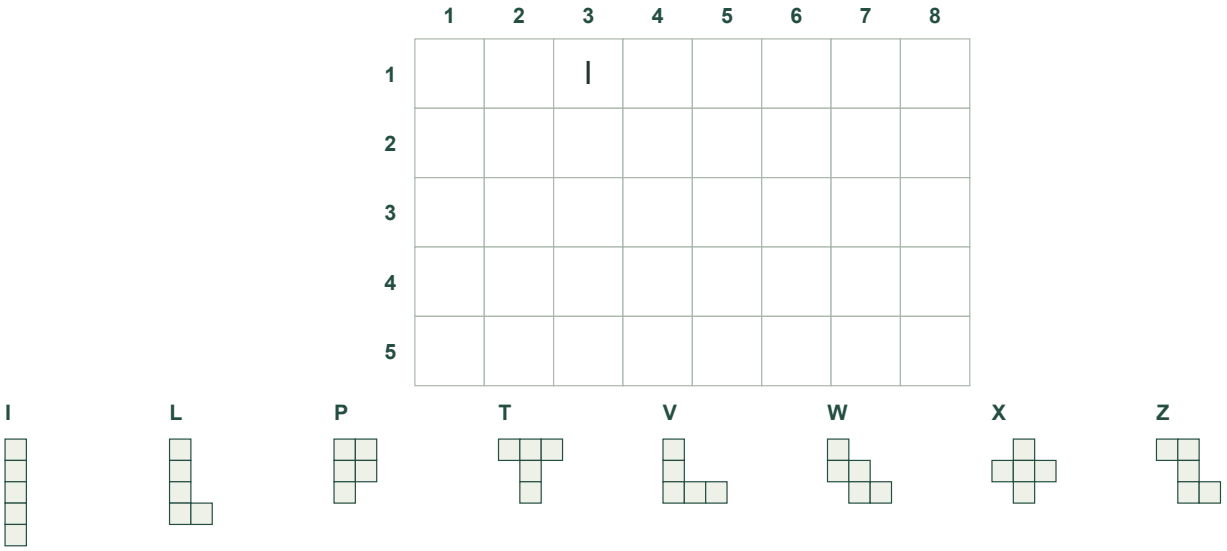
Materials. Pencil and gum; eventually reproduce the 8 silhouettes on gridded paper to cut them.

Rules

- Each room contains exactly five boxes bound by their sides. Use each of the 8 letters exactly once.
- Rotations and turns are allowed. A piece cannot be deformed or cut.
- Any box containing a letter must be covered by the part of this letter. The other boxes of the rectangle must also be covered.

Puzzle

The piece shapes are shown below the board.



Working space

Solution p. 123

The Locked Mosaic

Pentomino tiling / EXTREME

Completely paste the rectangle 5 × 10 with the given 10 pentominos, without overlap.

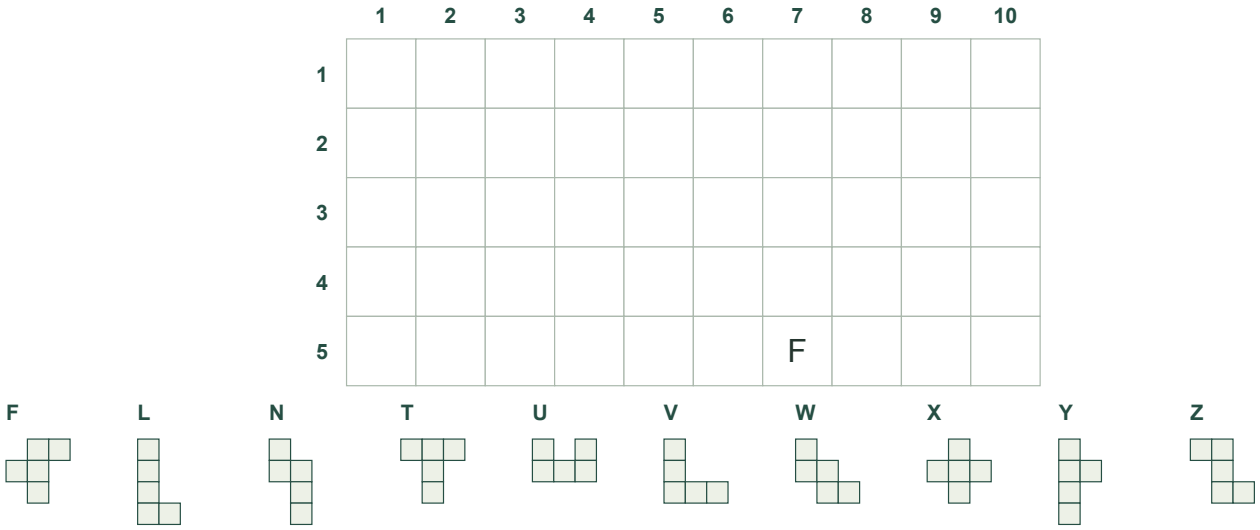
Materials. Pencil and gum; eventually reproduce the 10 silhouettes on gridded paper to cut them.

Rules

- Each room contains exactly five boxes bound by their sides. Use each of the 10 letters exactly once.
- Rotations and turns are allowed. A piece cannot be deformed or cut.
- Any box containing a letter must be covered by the part of this letter. The other boxes of the rectangle must also be covered.

Puzzle

The piece shapes are shown below the board.



Working space

Solution p. 125

The Stubborn Rectangle

Pentomino tiling / EXTREME

Completely paste the rectangle 5 × 10 with the given 10 pentominos, without overlap.

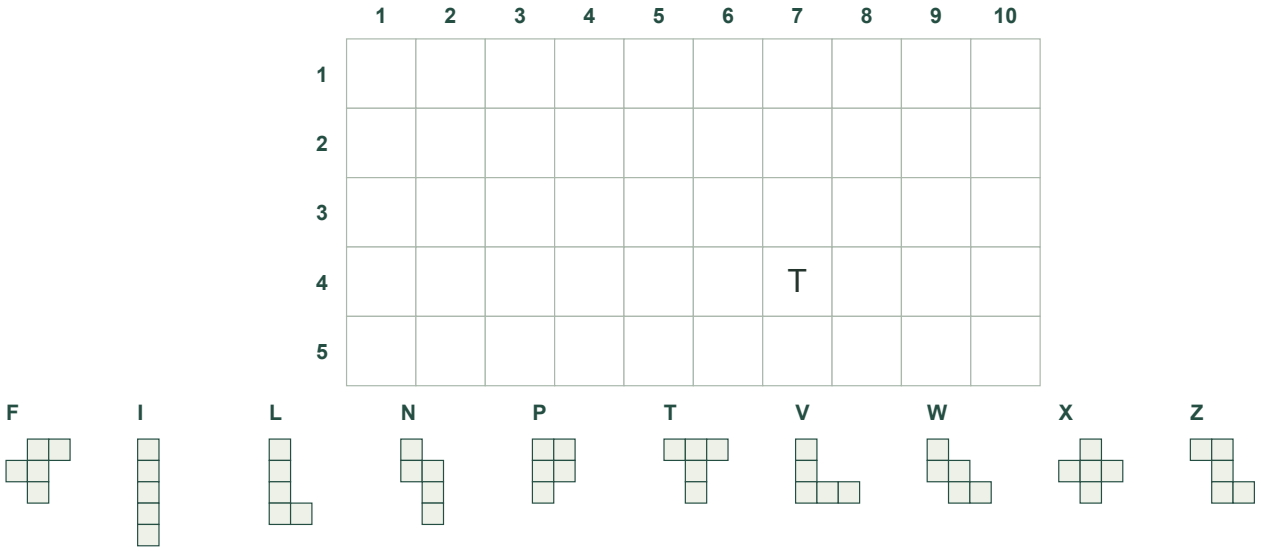
Materials. Pencil and gum; eventually reproduce the 10 silhouettes on gridded paper to cut them.

Rules

- Each room contains exactly five boxes bound by their sides. Use each of the 10 letters exactly once.
- Rotations and turns are allowed. A piece cannot be deformed or cut.
- Any box containing a letter must be covered by the part of this letter. The other boxes of the rectangle must also be covered.

Puzzle

The piece shapes are shown below the board.



Working space

Solution p. 127

The Final Tempo

Othello / Reversi / EXPERT

Black plays. Find his only best first shot and the final differential that he can guarantee against any optimal defense.

Materials. Pencil, gum; tray 8 × 8 and optional reversible pions.

Rules

- Tray 8 × 8; columns A to H from left to right, lines 1 to 8 from top to bottom. N = Black, B = White, · = empty box.

- A shot sets a pawn on an empty box and must frame at least one opposing pawn between this pawn and a friendly pawn, in a straight horizontal, vertical or diagonal line. All frames created are turned.

- Without a legal blow, you have to pass. If no camp can play, the game stops. Score: number of his pawns less number of opposing pawns; empty boxes do not count. The goal is to maximize this differential, not just to win.

Puzzle

Trait : Black. There are 8 empty boxes left.

For the evidence, also give the best defense after your first blow, then examine the other legal blows. No second person is necessary: analyze both sides on paper.

	A	B	C	D	E	F	G	H
1	B	N	N	N	·	N	B	N
2	B	B	N	N	·	B	B	N
3	B	N	N	N	B	N	B	N
4	N	N	N	N	B	B	B	N
5	N	B	B	N	B	B	B	N
6	·	B	B	B	N	N	B	N
7	·	·	B	B	B	B	B	N
8	·	B	·	B	·	B	B	N

Working space

Solution p. 129

The Deceptive Corner

Othello / Reversi / EXPERT

Black plays. Find his only best first shot and the final differential that he can guarantee against any optimal defense.

Materials. Pencil, gum; tray 8 × 8 and optional reversible pions.

Rules

- Tray 8 × 8; columns A to H from left to right, lines 1 to 8 from top to bottom. N = Black, B = White, · = empty box.

- A shot sets a pawn on an empty box and must frame at least one opposing pawn between this pawn and a friendly pawn, in a straight horizontal, vertical or diagonal line. All frames created are turned.

- Without a legal blow, you have to pass. If no camp can play, the game stops. Score: number of his pawns less number of opposing pawns; empty boxes do not count. The goal is to maximize this differential, not just to win.

Puzzle

Trait : Black. There are 8 empty boxes left.

For the evidence, also give the best defense after your first blow, then examine the other legal blows. No second person is necessary: analyze both sides on paper.

	A	B	C	D	E	F	G	H
1	B	B	B	B	B	B	·	N
2	B	B	N	B	B	B	N	B
3	·	B	B	B	B	N	B	·
4	N	N	N	N	N	B	B	B
5	N	N	N	B	B	N	B	B
6	N	N	B	N	B	B	B	B
7	N	N	B	B	·	·	B	B
8	N	N	B	N	·	·	·	B

Working space

Solution p. 131

Mobility in Reserve

Othello / Reversi / EXPERT

White plays. Find his only best first shot and the final differential that he can guarantee against any optimal defense.

Materials. Pencil, gum; tray 8 × 8 and optional reversible pions.

Rules

- Tray 8 × 8; columns A to H from left to right, lines 1 to 8 from top to bottom. N = Black, B = White, · = empty box.

- A shot sets a pawn on an empty box and must frame at least one opposing pawn between this pawn and a friendly pawn, in a straight horizontal, vertical or diagonal line. All frames created are turned.

- Without a legal blow, you have to pass. If no camp can play, the game stops. Score: number of his pawns less number of opposing pawns; empty boxes do not count. The goal is to maximize this differential, not just to win.

Puzzle

White to move. There are 9 empty squares.

For the evidence, also give the best defense after your first blow, then examine the other legal blows. No second person is necessary: analyze both sides on paper.

	A	B	C	D	E	F	G	H
1	N	N	N	N	N	N	B	·
2	·	N	N	N	N	N	N	N
3	·	N	B	B	N	B	N	N
4	B	N	B	B	B	N	B	·
5	B	N	N	N	B	N	N	B
6	B	N	B	N	B	N	B	B
7	B	N	B	B	N	N	·	B
8	·	N	·	·	B	N	·	B

Working space

Solution p. 132

The Defense Tree

Othello / Reversi / EXTREME

Black plays. Find his only best first shot and the final differential that he can guarantee against any optimal defense.

Materials. Pencil, gum; tray 8 × 8 and optional reversible pions.

Rules

- Tray 8 × 8; columns A to H from left to right, lines 1 to 8 from top to bottom. N = Black, B = White, · = empty box.

- A shot sets a pawn on an empty box and must frame at least one opposing pawn between this pawn and a friendly pawn, in a straight horizontal, vertical or diagonal line. All frames created are turned.

- Without a legal blow, you have to pass. If no camp can play, the game stops. Score: number of his pawns less number of opposing pawns; empty boxes do not count. The goal is to maximize this differential, not just to win.

Puzzle

Trait : Black. There are 10 empty boxes left.

For the evidence, also give the best defense after your first blow, then examine the other legal blows. No second person is necessary: analyze both sides on paper.

	A	B	C	D	E	F	G	H
1	B	N	N	·	B	·	·	N
2	B	N	N	B	·	·	·	N
3	·	B	B	N	N	N	N	N
4	N	B	B	N	B	N	B	N
5	B	N	N	B	N	N	B	N
6	·	N	B	B	B	N	B	N
7	·	B	N	N	N	N	N	·
8	B	B	N	N	N	N	N	N

Working space

Solution p. 134

Paths to a Majority

Othello / Reversi / EXTREME

White plays. Find his only best first shot and the final differential that he can guarantee against any optimal defense.

Materials. Pencil, gum; tray 8 × 8 and optional reversible pions.

Rules

- Tray 8 × 8; columns A to H from left to right, lines 1 to 8 from top to bottom. N = Black, B = White, · = empty box.

- A shot sets a pawn on an empty box and must frame at least one opposing pawn between this pawn and a friendly pawn, in a straight horizontal, vertical or diagonal line. All frames created are turned.

- Without a legal blow, you have to pass. If no camp can play, the game stops. Score: number of his pawns less number of opposing pawns; empty boxes do not count. The goal is to maximize this differential, not just to win.

Puzzle

Trait: White. There are 11 empty boxes left.

For the evidence, also give the best defense after your first blow, then examine the other legal blows. No second person is necessary: analyze both sides on paper.

	A	B	C	D	E	F	G	H
1	·	·	B	B	B	B	B	B
2	·	B	B	B	N	N	B	·
3	B	B	B	N	N	N	·	·
4	B	B	N	B	B	N	N	B
5	B	N	B	B	B	N	·	N
6	B	N	N	B	N	N	N	B
7	B	N	N	N	N	N	B	·
8	B	N	·	B	B	B	·	·

Working space

Solution p. 136

The Last Word

Othello / Reversi / EXTREME

White plays. Find his only best first shot and the final differential that he can guarantee against any optimal defense.

Materials. Pencil, gum; tray 8 × 8 and optional reversible pions.

Rules

- Tray 8 × 8; columns A to H from left to right, lines 1 to 8 from top to bottom. N = Black, B = White, · = empty box.

- A shot sets a pawn on an empty box and must frame at least one opposing pawn between this pawn and a friendly pawn, in a straight horizontal, vertical or diagonal line. All frames created are turned.

- Without a legal blow, you have to pass. If no camp can play, the game stops. Score: number of his pawns less number of opposing pawns; empty boxes do not count. The goal is to maximize this differential, not just to win.

Puzzle

Trait: White. There are 11 empty boxes left.

For the evidence, also give the best defense after your first blow, then examine the other legal blows. No second person is necessary: analyze both sides on paper.

	A	B	C	D	E	F	G	H
1	·	N	·	N	N	N	·	·
2	·	N	B	B	B	B	B	·
3	B	N	B	B	B	B	B	N
4	B	N	B	N	B	N	B	·
5	B	N	B	B	B	N	B	·
6	B	N	B	B	B	N	B	N
7	B	N	N	N	N	N	N	N
8	·	·	B	B	B	·	B	N

Working space

Solution p. 138

Two Workshops

Nim — pair subtraction / EXPERT

Find all the first shots that guarantee victory, and demonstrate strategy against any adverse response.

Materials. Pencil and paper; optional chips, all pile numbers are given.

Rules

- Original variant "pair subtraction". Each component is a pair of heaps A and B. The pairs remain independent.

- At each turn, choose exactly ONE pair, then remove 1, 2 or 3 chips from A alone, OR B alone, OR exactly the same number (1, 2 or 3) of both piles. No pile can become negative. Never remove two different pairs in the same round.

- The two sides alternate. The player without a legal blow loses: taking the last chips makes it win. There is neither pass nor rule misery. The tokens of the same pile are indistinguishable; the pairs and piles A/B are labeled.

Puzzle

You play first. To analyze alone, calculate the winning and losing positions of both sides.

Gives a complete list of the winners and a general proof of answer: only one cooperative suite is not enough.

Pair	Pile A	Pile B
1	8	11
2	10	7

Working space

Solution p. 140

The Broken Balance

Nim — pair subtraction / EXPERT

Find all the first shots that guarantee victory, and demonstrate strategy against any adverse response.

Materials. Pencil and paper; optional chips, all pile numbers are given.

Rules

- Original variant "pair subtraction". Each component is a pair of heaps A and B. The pairs remain independent.

- At each turn, choose exactly ONE pair, then remove 1, 2 or 3 chips from A alone, OR B alone, OR exactly the same number (1, 2 or 3) of both piles. No pile can become negative. Never remove two different pairs in the same round.

- The two sides alternate. The player without a legal blow loses: taking the last chips makes it win. There is neither pass nor rule misery. The tokens of the same pile are indistinguishable; the pairs and piles A/B are labeled.

Puzzle

You play first. To analyze alone, calculate the winning and losing positions of both sides.

Gives a complete list of the winners and a general proof of answer: only one cooperative suite is not enough.

Pair	Pile A	Pile B
1	12	9
2	7	13

Working space

Solution p. 142

Three Linked Fronts

Nim — pair subtraction / EXTREME

Find all the first shots that guarantee victory, and demonstrate strategy against any adverse response.

Materials. Pencil and paper; optional chips, all pile numbers are given.

Rules

- Original variant "pair subtraction". Each component is a pair of heaps A and B. The pairs remain independent.

- At each turn, choose exactly ONE pair, then remove 1, 2 or 3 chips from A alone, OR B alone, OR exactly the same number (1, 2 or 3) of both piles. No pile can become negative. Never remove two different pairs in the same round.

- The two sides alternate. The player without a legal blow loses: taking the last chips makes it win. There is neither pass nor rule misery. The tokens of the same pile are indistinguishable; the pairs and piles A/B are labeled.

Puzzle

You play first. To analyze alone, calculate the winning and losing positions of both sides.

Gives a complete list of the winners and a general proof of answer: only one cooperative suite is not enough.

Pair	Pile A	Pile B
1	17	19
2	14	18
3	20	15

Working space

Solution p. 144

The Great Pile Board

Nim — pair subtraction / EXTREME

Find all the first shots that guarantee victory, and demonstrate strategy against any adverse response.

Materials. Pencil and paper; optional chips, all pile numbers are given.

Rules

- Original variant "pair subtraction". Each component is a pair of heaps A and B. The pairs remain independent.

- At each turn, choose exactly ONE pair, then remove 1, 2 or 3 chips from A alone, OR B alone, OR exactly the same number (1, 2 or 3) of both piles. No pile can become negative. Never remove two different pairs in the same round.

- The two sides alternate. The player without a legal blow loses: taking the last chips makes it win. There is neither pass nor rule misery. The tokens of the same pile are indistinguishable; the pairs and piles A/B are labeled.

Puzzle

You play first. To analyze alone, calculate the winning and losing positions of both sides.

Gives a complete list of the winners and a general proof of answer: only one cooperative suite is not enough.

Pair	Pile A	Pile B
1	21	16
2	18	23
3	22	19

Working space

Solution p. 146

PART TWO

Solutions to the Great Solo Challenge

Answers and detailed explanations.

The Ninth Lock

Classic Sudoku / EXPERT

Answer

423715689 / 615893247 / 987624513 / 876532194 / 132469875 / 549187362 / 758246931 / 364951728 / 291378456

	1	2	3	4	5	6	7	8	9
1	4	2	3	7	1	5	6	8	9
2	6	1	5	8	9	3	2	4	7
3	9	8	7	6	2	4	5	1	3
4	8	7	6	5	3	2	1	9	4
5	1	3	2	4	6	9	8	7	5
6	5	4	9	1	8	7	3	6	2
7	7	5	8	2	4	6	9	3	1
8	3	6	4	9	5	1	7	2	8
9	2	9	1	3	7	8	4	5	6

Reasoning

1. Note the candidates in each box by crossing the line, column and region.

2. In the certificate below, units 1 to 9 are lines, 10 to 18 columns, 19 to 27 regions. Units 28 and 29, if present, are diagonals.

3. A nude subset of two or three cells reserves as many numbers: these figures are excluded from the other cells of the unit. An interaction limits one digit in the intersection of two units and excludes from the rest of the second.

4. Each hypothesis opens a separate branch. Never keep in the neighbouring branch a value found under an abandoned hypothesis. The contradictions and all the figures deduced appear in the tree of evidence.

5. The search finds exactly a complete grid. It runs through 3 states; the 1 contradictory branches are closed. Finally check each unit and each starting point.

First deductions

L3C3: only place for 7 in unit 3 → 7

L5C8: only place for 7 in unit 5 → 7

L6C6: only place for 7 in unit 6 → 7

L4C2: only place for 7 in unit 4 → 7

L7C8: only place for 3 in unit 7 → 3

L3C9: only place for 3 in unit 3 → 3

First assumptions tested. L1C9 = 8 or L1C9 = 9.

Common pitfalls

- Consolidate a temporary hypothesis with a demonstrated value.
- Use a subset of three boxes with four digits in the union.
- Forget a variant constraint or keep data from a refuted branch.

Verification

Exact enumeration, shutdown after two solutions, terminal verification of all units and cages.

The Hidden Crossroads

Classic Sudoku / EXTREME

Answer

745163829 / 123958746 / 986247351 / 269714538 / 418395672 / 537826194 / 374689215 / 691532487 / 852471963

	1	2	3	4	5	6	7	8	9
1	7	4	5	1	6	3	8	2	9
2	1	2	3	9	5	8	7	4	6
3	9	8	6	2	4	7	3	5	1
4	2	6	9	7	1	4	5	3	8
5	4	1	8	3	9	5	6	7	2
6	5	3	7	8	2	6	1	9	4
7	3	7	4	6	8	9	2	1	5
8	6	9	1	5	3	2	4	8	7
9	8	5	2	4	7	1	9	6	3

Reasoning

1. Note the candidates in each box by crossing the line, column and region.

2. In the certificate below, units 1 to 9 are lines, 10 to 18 columns, 19 to 27 regions. Units 28 and 29, if present, are diagonals.

3. A nude subset of two or three cells reserves as many numbers: these figures are excluded from the other cells of the unit. An interaction limits one digit in the intersection of two units and excludes from the rest of the second.

4. Each hypothesis opens a separate branch. Never keep in the neighbouring branch a value found under an abandoned hypothesis. The contradictions and all the figures deduced appear in the tree of evidence.

5. The search finds exactly a complete grid. It runs through 29 states; the 14 contradictory branches are closed. Finally check each unit and each starting index.

First deductions

L1C8: only candidate → 2

L5C9: only place for 2 in unit 5 → 2

L6C7: only place for 1 in unit 6 → 1

L3C9: only place for 1 in unit 18 → 1

L4C9: only place for 8 in unit 18 → 8

L9C9: only place for 3 in unit 18 → 3

First assumptions tested. L2C1 = 1 or L2C1 = 7.

Common pitfalls

- Consolidate a temporary hypothesis with a demonstrated value.
- Use a subset of three boxes with four digits in the union.
- Forget a variant constraint or keep data from a refuted branch.

Verification

Exact enumeration, shutdown after two solutions, terminal verification of all units and cages.

The Inverted Mesh

Classic Sudoku / EXPERT

Answer

345278916 / 791635824 / 862491537 / 529713468 / 673984152 / 184562379 / 218347695 / 937856241 / 456129783

	1	2	3	4	5	6	7	8	9
1	3	4	5	2	7	8	9	1	6
2	7	9	1	6	3	5	8	2	4
3	8	6	2	4	9	1	5	3	7
4	5	2	9	7	1	3	4	6	8
5	6	7	3	9	8	4	1	5	2
6	1	8	4	5	6	2	3	7	9
7	2	1	8	3	4	7	6	9	5
8	9	3	7	8	5	6	2	4	1
9	4	5	6	1	2	9	7	8	3

Reasoning

1. Note the candidates in each box by crossing the line, column and region.

2. In the certificate below, units 1 to 9 are lines, 10 to 18 columns, 19 to 27 regions. Units 28 and 29, if present, are diagonals.

3. A nude subset of two or three cells reserves as many numbers: these figures are excluded from the other cells of the unit. An interaction limits one digit in the intersection of two units and excludes from the rest of the second.

4. Each hypothesis opens a separate branch. Never keep in the neighbouring branch a value found under an abandoned hypothesis. The contradictions and all the figures deduced appear in the tree of evidence.

5. The search finds exactly a complete grid. It runs through 3 states; the 1 contradictory branches are closed. Finally check each unit and each starting point.

First deductions

L2C2: only place for 9 in unit 2 → 9

L3C9: only place for 7 in unit 3 → 7

L5C5: only place for 8 in unit 5 → 8

L6C9: only place for 9 in unit 6 → 9

L5C4: only place for 9 in unit 5 → 9

L5C2: only place for 7 in unit 5 → 7

First assumptions tested. L1C2 = 3 or L1C2 = 4.

Common pitfalls

- Consolidate a temporary hypothesis with a demonstrated value.
- Use a subset of three boxes with four digits in the union.
- Forget a variant constraint or keep data from a refuted branch.

Verification

Exact enumeration, shutdown after two solutions, terminal verification of all units and cages.

Branches of Silence

Classic Sudoku / EXTREME

Answer

762983145 / 395142876 / 481657329 / 146298753 / 859376412 / 273514968 / 618429537 / 924735681 / 537861294

	1	2	3	4	5	6	7	8	9
1	7	6	2	9	8	3	1	4	5
2	3	9	5	1	4	2	8	7	6
3	4	8	1	6	5	7	3	2	9
4	1	4	6	2	9	8	7	5	3
5	8	5	9	3	7	6	4	1	2
6	2	7	3	5	1	4	9	6	8
7	6	1	8	4	2	9	5	3	7
8	9	2	4	7	3	5	6	8	1
9	5	3	7	8	6	1	2	9	4

Reasoning

1. Note the candidates in each box by crossing the line, column and region.

2. In the certificate below, units 1 to 9 are lines, 10 to 18 columns, 19 to 27 regions. Units 28 and 29, if present, are diagonals.

3. A nude subset of two or three cells reserves as many numbers: these figures are excluded from the other cells of the unit. An interaction limits one digit in the intersection of two units and excludes from the rest of the second.

4. Each hypothesis opens a separate branch. Never keep in the neighbouring branch a value found under an abandoned hypothesis. The contradictions and all the figures deduced appear in the tree of evidence.

5. The search finds exactly a complete grid. It runs through 81 states; the 40 contradictory branches are closed. Finally check each unit and each starting index.

First deductions

L5C8: only candidate → 1

L1C9: only place for 5 in unit 1 → 5

L1C2: only place for 6 in unit 1 → 6

L1C7: only place for 1 in unit 1 → 1

L2C3: only place for 5 in unit 2 → 5

L2C9: only place for 6 in unit 2 → 6

First assumptions tested. L1C4 = 3 or L1C4 = 9.

Common pitfalls

- Consolidate a temporary hypothesis with a demonstrated value.

- Use a subset of three boxes with four digits in the union.

- Forget a variant constraint or keep data from a refuted branch.

Verification

Exact enumeration, shutdown after two solutions, terminal verification of all units and cages.

The Two Crossings

Diagonal Sudoku / EXPERT

Answer

716854392 / 893612754 / 245973168 / 371489625 / 624135879 / 589726413 / 138597246 / 962341587 / 457268931

	1	2	3	4	5	6	7	8	9
1	7	1	6	8	5	4	3	9	2
2	8	9	3	6	1	2	7	5	4
3	2	4	5	9	7	3	1	6	8
4	3	7	1	4	8	9	6	2	5
5	6	2	4	1	3	5	8	7	9
6	5	8	9	7	2	6	4	1	3
7	1	3	8	5	9	7	2	4	6
8	9	6	2	3	4	1	5	8	7
9	4	5	7	2	6	8	9	3	1

Reasoning

1. Note candidates in each box by crossing lines, columns and regions, diagonals.

2. In the certificate below, units 1 to 9 are lines, 10 to 18 columns, 19 to 27 regions. Units 28 and 29, if present, are diagonals.

3. A nude subset of two or three cells reserves as many numbers: these figures are excluded from the other cells of the unit. An interaction limits one digit in the intersection of two units and excludes from the rest of the second.

4. Each hypothesis opens a separate branch. Never keep in the neighbouring branch a value found under an abandoned hypothesis. The contradictions and all the figures deduced appear in the tree of evidence.

5. The search finds exactly one complete grid. It visits 5 states and rules out the 2 contradictory branches. Check each unit and starting clue.

First deductions

L9C1: only place for 4 in unit 9 → 4

L9C7: only place for 9 in unit 9 → 9

L9C2: only place for 5 in unit 9 → 5

L5C5: only place for 3 in unit 28 → 3

L4C1: only place for 3 in unit 4 → 3

L7C2: only place for 3 in unit 11 → 3

First assumptions tested. L3C7 = 1 or L3C7 = 2.

Common pitfalls

- Consolidate a temporary hypothesis with a demonstrated value.

- Use a subset of three boxes with four digits in the union.

- Forget a variant constraint or keep data from a refuted branch.

Verification

Exact enumeration, shutdown after two solutions, terminal verification of all units and cages.

The Stubborn Mirror

Diagonal Sudoku / EXTREME

Answer

679852413 / 432671859 / 815943672 / 356297148 / 284516937 / 791438265 / 548369721 / 123784596 / 967125384

	1	2	3	4	5	6	7	8	9
1	6	7	9	8	5	2	4	1	3
2	4	3	2	6	7	1	8	5	9
3	8	1	5	9	4	3	6	7	2
4	3	5	6	2	9	7	1	4	8
5	2	8	4	5	1	6	9	3	7
6	7	9	1	4	3	8	2	6	5
7	5	4	8	3	6	9	7	2	1
8	1	2	3	7	8	4	5	9	6
9	9	6	7	1	2	5	3	8	4

Reasoning

1. Note candidates in each box by crossing lines, columns and regions, diagonals.

2. In the certificate below, units 1 to 9 are lines, 10 to 18 columns, 19 to 27 regions. Units 28 and 29, if present, are diagonals.

3. A nude subset of two or three cells reserves as many numbers: these figures are excluded from the other cells of the unit. An interaction limits one digit in the intersection of two units and excludes from the rest of the second.

4. Each hypothesis opens a separate branch. Never keep in the neighbouring branch a value found under an abandoned hypothesis. The contradictions and all the figures deduced appear in the tree of evidence.

5. The search finds exactly a complete grid. It runs through 37 states; the 18 contradictory branches are closed. Finally, check each unit and each starting point.

First deductions

L1C7: only place for 4 in unit 1 → 4

L8C6: only place for 4 in unit 15 → 4

L9C9: only place for 4 in unit 28 → 4

L7C2: only place for 4 in unit 7 → 4

L7C9: only place for 1 in unit 7 → 1

Units 1 and 19 overlap. In unit 1, 6 must lie in the overlap. Remove it from L3C1, L3C2, L3C3.

First assumptions tested. L6C6 = 3 or L6C6 = 8.

Common pitfalls

- Consolidate a temporary hypothesis with a demonstrated value.
- Use a subset of three boxes with four digits in the union.
- Forget a variant constraint or keep data from a refuted branch.

Verification

Exact enumeration, shutdown after two solutions, terminal verification of all units and cages.

The Living Border

Irregular Sudoku / EXPERT

Answer

564912378 / 783694125 / 128537946 / 215873469 / 379465281 / 496281537 / 637149852 / 942758613 / 851326794

	1	2	3	4	5	6	7	8	9
1	5	6	4	9	1	2	3	7	8
2	7	8	3	6	9	4	1	2	5
3	1	2	8	5	3	7	9	4	6
4	2	1	5	8	7	3	4	6	9
5	3	7	9	4	6	5	2	8	1
6	4	9	6	2	8	1	5	3	7
7	6	3	7	1	4	9	8	5	2
8	9	4	2	7	5	8	6	1	3
9	8	5	1	3	2	6	7	9	4

Reasoning

1. Note the candidates in each box by crossing the line, column and region.

2. In the certificate below, units 1 to 9 are lines, 10 to 18 columns, 19 to 27 regions. Units 28 and 29, if present, are diagonals.

3. A nude subset of two or three cells reserves as many numbers: these figures are excluded from the other cells of the unit. An interaction limits one digit in the intersection of two units and excludes from the rest of the second.

4. Each hypothesis opens a separate branch. Never keep in the neighbouring branch a value found under an abandoned hypothesis. The contradictions and all the figures deduced appear in the tree of evidence.

5. The search finds exactly a complete grid. It runs through 3 states; the 1 contradictory branches are closed. Finally check each unit and each starting point.

First deductions

L1C6: only place for 2 in unit 1 → 2

L2C8: only place for 2 in unit 2 → 2

L7C7: only place for 8 in unit 16 → 8

L3C3: only place for 8 in unit 12 → 8

L2C4: only candidate → 6

L3C4: only candidate → 5

First assumptions tested. L2C1 = 1 or L2C1 = 7.

Common pitfalls

- Consolidate a temporary hypothesis with a demonstrated value.
- Use a subset of three boxes with four digits in the union.
- Forget a variant constraint or keep data from a refuted branch.

Verification

Exact enumeration, shutdown after two solutions, terminal verification of all units and cages.

The Nine Territories

Irregular Sudoku / EXTREME

Answer

486753129 / 924875631 / 531962748 / 613294857 / 759381264 / 847126593 / 298547316 / 165438972 / 372619485

	1	2	3	4	5	6	7	8	9
1	4	8	6	7	5	3	1	2	9
2	9	2	4	8	7	5	6	3	1
3	5	3	1	9	6	2	7	4	8
4	6	1	3	2	9	4	8	5	7
5	7	5	9	3	8	1	2	6	4
6	8	4	7	1	2	6	5	9	3
7	2	9	8	5	4	7	3	1	6
8	1	6	5	4	3	8	9	7	2
9	3	7	2	6	1	9	4	8	5

Reasoning

1. Note the candidates in each box by crossing the line, column and region.

2. In the certificate below, units 1 to 9 are lines, 10 to 18 columns, 19 to 27 regions. Units 28 and 29, if present, are diagonals.

3. A nude subset of two or three cells reserves as many numbers: these figures are excluded from the other cells of the unit. An interaction limits one digit in the intersection of two units and excludes from the rest of the second.

4. Each hypothesis opens a separate branch. Never keep in the neighbouring branch a value found under an abandoned hypothesis. The contradictions and all the figures deduced appear in the tree of evidence.

5. The search finds exactly a complete grid. It runs through 27 states; the 13 contradictory branches are closed. Finally check each unit and each starting point.

First deductions

L9C1: only place for 3 in unit 10 → 3

L7C5: only place for 4 in unit 26 → 4

Units 3 and 21 overlap. In unit 3, 7 must lie in the overlap. Remove it from L2C6.

Units 8 and 27 overlap. In unit 8, 2 must lie in the overlap. Remove it from L9C8, L9C9.

Units 13 and 23 overlap. In unit 13, 3 must lie in the overlap. Remove it from L6C6, L6C7.

Units 23 and 5 overlap. In unit 23, 8 must lie in the overlap. Remove it from L5C8, L5C9.

First assumptions tested. L4C1 = 5 or L4C1 = 6.

Common pitfalls

- Consolidate a temporary hypothesis with a demonstrated value.
- Use a subset of three boxes with four digits in the union.
- Forget a variant constraint or keep data from a refuted branch.

Verification

Exact enumeration, shutdown after two solutions, terminal verification of all units and cages.

Captive Sums

Killer Sudoku / EXPERT

Answer

861472395 / 537916428 / 294835176 / 415283769 / 678149532 / 923567841 / 782654913 / 146398257 / 359721684

	1	2	3	4	5	6	7	8	9
1	20 8	6	1	13 4	7	2	12 3	9	22 5
2	5	18 3	24 7	9	11 1	6	4	2	8
3	2	9	4	8	16 3	5	1	7	17 6
4	12 4	1	16 5	2	21 8	3 3	7	14 6	9
5	15 6	7	8	1	4	9	13 5	3	2
6	9	10 2	3	5	18 6	7	8	4	1
7	15 7	8	8 2	18 6	5	12 4	17 9	1	14 3
8	19 1	4	6	3	9 9	8	2	5	7
9	3 3	5	9	7	2	15 1	6	8	4

Solid lines mark regions. Dashed lines and small numbers mark cages and their totals.

Reasoning

1. Note the candidates in each box by crossing the line, column and region and amounts of cages.

2. In the certificate below, units 1 to 9 are lines, 10 to 18 columns, 19 to 27 regions. Units 28 and 29, if present, are diagonals.

3. A nude subset of two or three cells reserves as many numbers: these figures are excluded from the other cells of the unit. An interaction limits one digit in the intersection of two units and excludes from the rest of the second.

4. Each hypothesis opens a separate branch. Never keep in the neighbouring branch a value found under an abandoned hypothesis. The contradictions and all the figures deduced appear in the tree of evidence.

5. The search finds exactly a complete grid. It runs through 9 states; the 4 contradictory branches are closed. Finally check each unit and each starting point.

First deductions

L4C6: only candidate → 3

L8C5: only candidate → 9

L9C1: only candidate → 3

Nude sub-assembly in Unit 10: L5C1, L6C1 reserve [6, 9]; remove these numbers from L2C1, L3C1, L7C1, L8C1.

L7C1: only candidate → 7

L7C2: only candidate → 8

First assumptions tested. L2C3 = 7 or L2C3 = 9.

Common pitfalls

- Consolidate a temporary hypothesis with a demonstrated value.

- Use a subset of three boxes with four digits in the union.

- Forget a variant constraint or keep data from a refuted branch.

Verification

Exact enumeration, shutdown after two solutions, terminal verification of all units and cages.

Unyielding Totals

Killer Sudoku / EXTREME

Answer

637412598 / 981375624 / 245896731 / 528961473 / 174238965 / 396754182 / 712589346 / 463127859 / 859643217

	1	2	3	4	5	6	7	8	9
1	9 6	3	14 7	4	1	2	13 5	22 9	8
2	16 9	18 8	1	27 3	7	19 5	6	2	4
3	2	4	5	8	18 9	6	7	20 3	1
4	5	17 2	8	9	6	1	4	7	8 3
5	13 1	7	10 4	18 2	3	21 8	9	6	5
6	3	9	6	7	5	4	1	19 8	2
7	19 7	19 1	8 2	5	27 8	9	3	4	15 6
8	4	6	3	1	12 2	7	11 8	5	9
9	8	5 5	9	6	4	3	2	1	7 7

Solid lines mark regions. Dashed lines and small numbers mark cages and their totals.

Reasoning

1. Note the candidates in each box by crossing the line, column and region and amounts of cages.

2. In the certificate below, units 1 to 9 are lines, 10 to 18 columns, 19 to 27 regions. Units 28 and 29, if present, are diagonals.

3. A nude subset of two or three cells reserves as many numbers: these figures are excluded from the other cells of the unit. An interaction limits one digit in the intersection of two units and excludes from the rest of the second.

4. Each hypothesis opens a separate branch. Never keep in the neighbouring branch a value found under an abandoned hypothesis. The contradictions and all the figures deduced appear in the tree of evidence.

5. The search finds exactly a complete grid. It runs through 127 states; the 63 contradictory branches are closed. Finally, check each unit and each starting point.

First deductions

L8C9: only candidate → 9

L9C2: only candidate → 5

L9C9: only candidate → 7

Nude subset in unit 18: L4C9, L5C9 reserve [3, 5]; remove these numbers from L1C9, L2C9, L3C9, L6C9.

L1C8: only candidate → 9

L6C9: only place for 2 in unit 18 → 2

First assumptions tested. L1C7 = 5 or L1C7 = 6.

Common pitfalls

- Consolidate a temporary hypothesis with a demonstrated value.

- Use a subset of three boxes with four digits in the union.

- Forget a variant constraint or keep data from a refuted branch.

Verification

Exact enumeration, shutdown after two solutions, terminal verification of all units and cages.

The Eight Inks

Advanced Mastermind / EXPERT

Answer

Code: 70730.

Reasoning

1. Separates two constraints for each line: the number of equal positions per position, and then the total of common occurrences. This total is well placed + wrong placed; for each number, the minimum of the two multiplicities is taken.

2. To cut down the work, ignore the positions for now. Let n0,…,n7 be the counts of the eight digits. Their sum is 5.

3. Proposal 02553 requires min(n0, 1) + min(n2, 1) + min(n3, 1) + min(n5, 2) = 2.

4. Proposal 62532 requires min(n2, 2) + min(n3, 1) + min(n5, 1) + min(n6, 1) = 1.

5. Proposal 05062 requires min(n0, 2) + min(n2, 1) + min(n5, 1) + min(n6, 1) = 2.

6. Proposal 73375 requires min(n3, 2) + min(n5, 1) + min(n7, 2) = 3.

7. These limitations of occurrences alone leave 5 multi-unit(s).Here they are, written in ascending order: 00337; 00377; 01577; 04577; 05777.

8. Multiassembly 00337 : after successive application of the numbers of property placed, there remains 12 → 5 → 2 → 0 arrangement(s).

9. Multiassembly 00377 : after successive application of the numbers of property placed, there remains 15 → 3 → 1 → 1 arrangement(s).

10. Multi ensemble 01577 : after successive application of the numbers of property placed, there is left 30 → 0 → 0 → 0 arrangement(s).

11. Multi ensemble 04577 : after successive application of the numbers of property placed, there remains 30 → 0 → 0 → 0 arrangement(s).

12. Multiassembly 05777 : after successive application of the numbers of property placed, there remains 10 → 0 → 0 → 0 arrangement(s).

13. So we can make two separate tables, occurrences and then places. Only one line in the second table keeps an arrangement; that's the code announced. This separation avoids blindly browsing all the ordered words.

14. Starts with the 32.768 words ordered length 5 out of 0 to 7 and do not remove the initial zeros or the words with repetition.

15. Index 1, proposal 02553: retains exactly the configurations with 0 position equal(s) and 2 common occurrence(s). There are 7.051 candidates left.

16. Index 2, proposal 62532: keeps exactly the configurations with 1 position equal(s) and 1 common occurrence(s). There are 319 candidates left.

17. Index 3, proposal 05062: retains exactly the configurations with 0 position equal(s) and 2 common occurrence(s). There are 16 candidates left.

18. Index 4, proposal 73375: retains exactly the configurations with 1 position equal(s) and 3 common occurrence(s). There is 1 candidate remaining.

19. In index 1, candidate 00000 would give 1 well placed and 0 badly placed, instead of 0 and 2: it is eliminated.

20. In Index 2, candidate 10005 would give 0 well placed and 1 ill placed, instead of 1 and 0: it is eliminated.

21. In index 3, candidate 10002 would give 2 well placed and 1 ill placed, instead of 0 and 2: it is eliminated.

22. This filtering can be done on paper in branches: first fixes the possible occurrences, then affects their places and cuts a branch as soon as a line exceeds its number of places or can no longer reach its common total. You must not choose a candidate only because it satisfies the last line.

23. Final check against 02553: 0 well placed, 2 wrong placed.

24. Final check against 62532: 1 well placed, 0 wrong placed.

25. Final check against 05062: 0 well placed, 2 wrong placed.

26. Final check against 73375: 1 well placed, 2 badly placed.

27. The exhaustive check takes all possible words and has exactly one solution. The numbers are therefore determined at all places; it is not just a compatible code found at random.

Common pitfalls

- Consolidate wrongly placed and figure absent.

- Count twice an occurrence of the code when the proposal repeats a number.

- Prohibit repetitions or the initial zero without the rules requiring it.

Verification

Complete listing of all codes; calculation independent of each score; deletion of redundant indices.

The Silent Lock

Advanced Mastermind / EXPERT

Answer

Code: 32752.

Reasoning

1. Separates two constraints for each line: the number of equal positions per position, and then the total of common occurrences. This total is well placed + wrong placed; for each number, the minimum of the two multiplicities is taken.

2. To cut down the work, ignore the positions for now. Let n0,…,n7 be the counts of the eight digits. Their sum is 5.

3. Proposal 22543 requires min(n2, 2) + min(n3, 1) + min(n4, 1) + min(n5, 1) = 4.

4. Proposal 41050 requires min(n0, 2) + min(n1, 1) + min(n4, 1) + min(n5, 1) = 1.

5. Proposal 70302 requires min(n0, 2) + min(n2, 1) + min(n3, 1) + min(n7, 1) = 3.

6. Proposal 37433 requires min(n3, 3) + min(n4, 1) + min(n7, 1) = 2.

7. These limitations of occurrences alone leave 1 multi set(s).Here they are, written in ascending order: 22357.

8. Multi ensemble 22357 : after successive application of the numbers of property placed, there remains 26 → 6 → 2 → 1 arrangement(s).

9. So we can make two separate tables, occurrences and then places. Only one line in the second table keeps an arrangement; that's the code announced. This separation avoids blindly browsing all the ordered words.

10. Starts with the 32.768 words ordered length 5 out of 0 to 7 and do not remove the initial zeros or the words with repetition.

11. Index 1, proposal 22543: keeps exactly the configurations with 1 position equal(s) and 4 common occurrence(s). There are 652 candidates left.

12. Index 2, proposal 41050: retains exactly the configurations with 1 position equal(s) and 1 common occurrence(s). There are 39 candidates left.

13. Index 3, proposal 70302: keeps exactly the configurations with 1 position equal(s) and 3 common occurrence(s). There are 4 candidates left.

14. Index 4, proposal 37433: retains exactly the configurations with 1 position equal(s) and 2 common occurrence(s). There are 1 candidate(s).

15. In Index 1, candidate 00000 would give 0 well placed and 0 badly placed, instead of 1 and 3: it is eliminated.

16. In Index 2, candidate 02234 would give 0 well placed and 2 badly placed, instead of 1 and 0: it is eliminated.

17. In index 3, candidate 23252 would give 1 well placed and 1 ill placed, instead of 1 and 2: it is eliminated.

18. This filtering can be done on paper in branches: first fixes the possible occurrences, then affects their places and cuts a branch as soon as a line exceeds its number of places or can no longer reach its common total. You must not choose a candidate only because it satisfies the last line.

19. Final check against 22543: 1 well placed, 3 badly placed.

20. Final check against 41050: 1 well placed, 0 wrong placed.

21. Final check against 70302 : 1 well placed, 2 wrong placed.

22. Final check against 37433 : 1 well placed, 1 wrong placed.

23. The exhaustive check takes all possible words and has exactly one solution. The numbers are therefore determined at all places; it is not just a compatible code found at random.

Common pitfalls

- Consolidate wrongly placed and figure absent.

- Count twice an occurrence of the code when the proposal repeats a number.

- Prohibit repetitions or the initial zero without the rules requiring it.

Verification

Complete listing of all codes; calculation independent of each score; deletion of redundant indices.

Six Slots Without an Echo

Advanced Mastermind / EXTREME

Answer

Code: 331037.

Reasoning

1. Separates two constraints for each line: the number of equal positions per position, and then the total of common occurrences. This total is well placed + wrong placed; for each number, the minimum of the two multiplicities is taken.

2. To cut down the work, ignore the positions for now. Let n0,…,n7 be the counts of the eight digits. Their sum is 6.

3. Proposal 345125 requires min(n1, 1) + min(n2, 1) + min(n3, 1) + min(n4, 1) + min(n5, 2) = 2.

4. Proposal 073421 requires min(n0, 1) + min(n1, 1) + min(n2, 1) + min(n3, 1) + min(n4, 1) + min(n7, 1) = 4.

5. Proposal 573237 requires min(n2, 1) + min(n3, 2) + min(n5, 1) + min(n7, 2) = 3.

6. Proposal 531562 requires min(n1, 1) + min(n2, 1) + min(n3, 1) + min(n5, 2) + min(n6, 1) = 2.

7. These limitations of occurrences alone leave 16 multi-group(s).Here they are, written in ascending order: 000237; 001277; 001337; 001377; 002237; 011277; 011337; 011377; 012277; 012777; 013337; 013777; 022237; 024677; 033467; 034677.

8. Multi ensemble 000237: after successive application of the numbers of property placed, there remains 32 → 13 → 1 → 0 arrangement(s).

9. Multi ensemble 001277 : after successive application of the numbers of property placed, there remains 48 → 10 → 0 → 0 arrangement(s).

10. Multi ensemble 001337 : after successive application of the numbers of property placed, there remains 66 → 28 → 2 → 0 arrangement(s).

11. Multi ensemble 001377 : after successive application of the numbers of property placed, there remains 48 → 19 → 1 → 0 arrangement(s).

12. Multi ensemble 002237 : after successive application of the numbers of property placed, there remains 66 → 15 → 2 → 0 arrangement(s).

13. Multi ensemble 011277 : after successive application of the numbers of property placed, there remains 66 → 21 → 0 → 0 arrangement(s).

14. Multi ensemble 011337 : after successive application of the numbers of property placed, there remains 72 → 24 → 2 → 0 arrangement(s).

15. Multi ensemble 011377 : after successive application of the numbers of property placed, there remains 66 → 23 → 3 → 0 arrangement(s).

16. Multi ensemble 012277 : after successive application of the numbers of property placed, there remains 66 → 10 → 0 → 0 arrangement(s).

17. Multi ensemble 012777 : after successive application of the numbers of property placed, there is left 32 → 6 → 0 → 0 arrangement(s).

18. Multi ensemble 013337 : after successive application of the numbers of property placed, there is left 56 → 21 → 3 → 1 arrangement(s).

19. Multi ensemble 013777 : after successive application of the numbers of property placed, there is left 32 → 11 → 1 → 0 arrangement(s).

20. Multi ensemble 022237 : after successive application of the numbers of property placed, there is left 56 → 7 → 1 → 0 arrangement(s).

21. Multi ensemble 024677 : after successive application of the numbers of property placed, there remains 96 → 39 → 4 → 0 arrangement(s).

22. Multi ensemble 033467 : after successive application of the numbers of property placed, there remains 132 → 58 → 3 → 0 arrangement(s).

23. Multi ensemble 034677 : after successive application of the numbers of property placed, there remains 96 → 45 → 4 → 0 arrangement(s).

24. So we can make two separate tables, occurrences and then places. Only one line in the second table keeps an arrangement; that's the code announced. This separation avoids blindly browsing all the ordered words.

25. Starts with the 262.144 ordered words of length 6 out of 0 to 7. The initial zeros or words with repetition are not removed.

26. Index 1, proposal 345125: retains exactly the configurations with 1 position equal(s) and 2 common occurrence(s). There are 26,034 candidates left.

27. Index 2, proposal 073421: retains exactly the configurations with 0 position level(s) and 4 common occurrence(s). There are 2,490 candidates left.

28. Index 3, proposal 573237: retains exactly the configurations with 2 positions and 3 common occurrence(s). There are 65 candidates left.

29. Index 4, proposal 531562: retains exactly the configurations with 2 positions and 2 common occurrence(s). There are 1 candidate(s) left.

30. In index 1, the candidate 000000 would give 0 well placed and 0 badly placed, instead of 1 and 1: it is eliminated.

31. In index 2, candidate 000015 would give 1 well placed and 1 ill placed, instead of 0 and 4: it is eliminated.

32. In index 3, candidate 100137 would give 2 well placed and 0 badly placed, instead of 2 and 1: it is eliminated.

33. This filtering can be done on paper in branches: first fixes the possible occurrences, then affects their places and cuts a branch as soon as a line exceeds its number of places or can no longer reach its common total. You must not choose a candidate only because it satisfies the last line.

34. Final check against 345125: 1 well placed, 1 wrong placed.

35. Final check against 073421 : 0 well placed, 4 wrong placed.

36. Final check against 573237: 2 well placed, 1 wrong placed.

37. Final check against 531562: 2 well placed, 0 wrong placed.

38. The exhaustive check takes all possible words and has exactly one solution. The numbers are therefore determined at all places; it is not just a compatible code found at random.

Common pitfalls

- Consolidate wrongly placed and figure absent.

- Count twice an occurrence of the code when the proposal repeats a number.

- Prohibit repetitions or the initial zero without the rules requiring it.

Verification

Complete listing of all codes; calculation independent of each score; deletion of redundant indices.

The Chamber of Duplicates

Advanced Mastermind / EXTREME

Answer

Code: 140633.

Reasoning

1. Separates two constraints for each line: the number of equal positions per position, and then the total of common occurrences. This total is well placed + wrong placed; for each number, the minimum of the two multiplicities is taken.

2. To cut down the work, ignore the positions for now. Let n0,…,n7 be the counts of the eight digits. Their sum is 6.

3. Proposal 474563 requires min(n3, 1) + min(n4, 2) + min(n5, 1) + min(n6, 1) + min(n7, 1) = 3.

4. Proposal 013237 requires min(n0, 1) + min(n1, 1) + min(n2, 1) + min(n3, 2) + min(n7, 1) = 4.

5. Proposal 163003 requires min(n0, 2) + min(n1, 1) + min(n3, 2) + min(n6, 1) = 5.

6. Proposal 635050 requires min(n0, 2) + min(n3, 1) + min(n5, 2) + min(n6, 1) = 3.

7. Proposal 042546 requires min(n0, 1) + min(n2, 1) + min(n4, 2) + min(n5, 1) + min(n6, 1) = 3.

8. These limitations of occurrences alone leave 1 multiset(s).Here they are, written in ascending order: 013346.

9. Multi ensemble 013346 : after successive application of the numbers of property placed, there remains 144 → 64 → 19 → 5 → 1 arrangement(s).

10. So we can make two separate tables, occurrences and then places. Only one line in the second table keeps an arrangement; that's the code announced. This separation avoids blindly browsing all the ordered words.

11. Starts with the 262.144 ordered words of length 6 out of 0 to 7. The initial zeros or words with repetition are not removed.

12. Index 1, proposal 474563: retains exactly the configurations with 1 position equal(s) and 3 common occurrence(s). There are 42,344 candidates left.

13. Index 2, proposal 013237: retains exactly the configurations with 1 position equal(s) and 4 common occurrence(s). There are 3,996 candidates left.

14. Index 3, proposal 163003: retains exactly the configurations with 2 positions and 5 common occurrence(s). 58 candidates remain.

15. Index 4, proposal 635050: retains exactly the configurations with 0 position equal(s) and 3 common occurrence(s). There are 5 candidates left.

16. Index 5, proposal 042546: retains exactly the configurations with 1 position equal(s) and 3 common occurrence(s). There are 1 candidate(s) left.

17. In index 1, the candidate 000000 would give 0 well placed and 0 badly placed, instead of 1 and 2: it is eliminated.

18. In index 2, candidate 000364 would give 1 well placed and 1 ill placed, instead of 1 and 3: it is eliminated.

19. In Index 3, candidate 001473 would give 1 well placed and 3 badly placed, instead of 2 and 3: it is eliminated.

20. This filtering can be done on paper in branches: first fixes the possible occurrences, then affects their places and cuts a branch as soon as a line exceeds its number of places or can no longer reach its common total. You must not choose a candidate only because it satisfies the last line.

21. Final check against 474563: 1 well placed, 2 badly placed.

22. Final check against 013237: 1 well placed, 3 badly placed.

23. Final check against 163003: 2 well placed, 3 badly placed.

24. Final check against 635050: 0 well placed, 3 badly placed.

25. Final check against 042546: 1 well placed, 2 badly placed.

26. The exhaustive check takes all possible words and has exactly one solution. The numbers are therefore determined at all places; it is not just a compatible code found at random.

Common pitfalls

- Consolidate wrongly placed and figure absent.

- Count twice an occurrence of the code when the proposal repeats a number.

- Prohibit repetitions or the initial zero without the rules requiring it.

Verification

Complete listing of all codes; calculation independent of each score; deletion of redundant indices.

The Twin Carries

Cryptarithm / EXPERT

Answer

A = 1, B = 3, C = 9, D = 5, E = 6, F = 7, G = 2, H = 4, I = 0, J = 8

5573 + 8462 = 14035

6707 + 8669 = 15376

Reasoning

1. Each column carries a hold of 0 or 1. For an addition of two numbers to four digits, a possible fifth position of the result is a final hold, so is 1.

2. Write a column equation, from right to left: first-term digit + second-time digit + incoming hold = result digit + 10 × outgoing hold. The same letter must keep its value in both additions.

3. Addition 1, column 1: B + G + r1_0 = D + 10 × r1_1.

4. Addition 1, column 2: F + E + r1_1 = B + 10 × r1_2.

5. Addition 1, column 3: D + H + r1_2 = I + 10 × r1_3.

6. Addition 1, column 4: D + J + r1_3 = H + 10 × r1_4.

7. Addition 2, column 1: F + C + r2_0 = E + 10 × r2_1.

8. Addition 2, column 2: I + E + r2_1 = F + 10 × r2_2.

9. Addition 2, column 3: F + E + r2_2 = B + 10 × r2_3.

10. Addition 2, column 4: E + J + r2_3 = D + 10 × r2_4.

11. In these equations, r1_0 and r2_0 are zero. Each other retention is 0 or 1; the same letter always refers to the same number.

12. Explore possible pairs of digits in the column of units, calculate the restraint and bar any branch giving the same number to two different letters. Repeat for tens, hundreds and thousands; a letter already affected no longer receives any new choice.

13. The first addition admits 16 partial assignments respecting these constraints. After assigning letters still absent to free numbers, 16 substitutions are checked against the second addition.

14. Crossing the two additions is essential: the entire substitution must be transported, not separately resolve each line and then juxtapose two incompatible responses.

15. Addition 1: DDFB + JHEG = AHIBD, i.e. 5573 + 8462 = 14035.

16. Column 1 from the right: 3 + 2 + carry 0 = 5. Write 5 and carry 0.

17. Column 2 from the right: 7 + 6 + carry 0 = 13. Write 3 and carry 1.

18. Column 3 from the right: 5 + 4 + carry 1 = 10. Write 0 and carry 1.

19. Column 4 from the right: 5 + 8 + carry 1 = 14. Write 4 and carry 1.

20. Addition 2: EFIF + JEEC = ADBFE, i.e. 6707 + 8669 = 15376.

21. Column 1 from the right: 7 + 9 + carry 0 = 16. Write 6 and carry 1.

22. Column 2 from the right: 0 + 6 + carry 1 = 7. Write 7 and carry 0.

23. Column 3 from the right: 7 + 6 + carry 0 = 13. Write 3 and carry 1.

24. Column 4 from the right: 6 + 8 + carry 1 = 15. Write 5 and carry 1.

25. False track of the first addition: A=1, B=2, D=7, E=9, F=3, G=5, H=6, I=4, J=8. It does not extend into any substitution that satisfies the second addition; keeping only an exact first sum would therefore be insufficient.

26. The solver lists all the deductions and all the injection assignments compatible with the first addition, then tests the second without approximation. Only one complete substitution remains.

Common pitfalls

- Attribute a number to a letter forgetting that it is already taken.

- Allow an initial zero.

- Forget to postpone a deduction or change the value of a letter between the two additions.

Verification

Complete search by columns, binary deductions, rotation of unallocated numbers and arithmetic control of both equals.

The Second Addition

Cryptarithm / EXPERT

Answer

A = 0, B = 9, C = 7, D = 1, E = 3, F = 8, G = 5, H = 6, I = 4, J = 2

1995 + 5523 = 7518

5748 + 8657 = 14405

Reasoning

1. Each column carries a hold of 0 or 1. For an addition of two numbers to four digits, a possible fifth position of the result is a final hold, so is 1.

2. Write a column equation, from right to left: first-term digit + second-time digit + incoming hold = result digit + 10 × outgoing hold. The same letter must keep its value in both additions.

3. Addition 1, column 1: G + E + r1_0 = F + 10 × r1_1.

4. Addition 1, column 2: B + J + r1_1 = D + 10 × r1_2.

5. Addition 1, column 3: B + G + r1_2 = G + 10 × r1_3.

6. Addition 1, column 4: D + G + r1_3 = C + 10 × r1_4.

7. Addition 2, column 1: F + C + r2_0 = G + 10 × r2_1.

8. Addition 2, column 2: I + G + r2_1 = A + 10 × r2_2.

9. Addition 2, column 3: C + H + r2_2 = I + 10 × r2_3.

10. Addition 2, column 4: G + F + r2_3 = I + 10 × r2_4.

11. In these equations, r1_0 and r2_0 are zero. Each other retention is 0 or 1; the same letter always refers to the same number.

12. Explore possible pairs of digits in the column of units, calculate the restraint and bar any branch giving the same number to two different letters. Repeat for tens, hundreds and thousands; a letter already affected no longer receives any new choice.

13. The first addition admits 39 partial assignments respecting these constraints. After assigning letters still absent to free numbers, 234 substitutions are checked against the second addition.

14. Crossing the two additions is essential: the entire substitution must be transported, not separately resolve each line and then juxtapose two incompatible responses.

15. Addition 1: DBBG + GGJE = CGDF, i.e. 1995 + 5523 = 7518.

16. Column 1 from the right: 5 + 3 + carry 0 = 8. Write 8 and carry 0.

17. Column 2 from the right: 9 + 2 + carry 0 = 11. Write 1 and carry 1.

18. Column 3 from the right: 9 + 5 + carry 1 = 15. Write 5 and carry 1.

19. Column 4 from the right: 1 + 5 + carry 1 = 7. Write 7 and carry 0.

20. Addition 2: GCIF + FHGC = DIIAG, 5748 + 8657 = 14405.

21. Column 1 from the right: 8 + 7 + carry 0 = 15. Write 5 and carry 1.

22. Column 2 from the right: 4 + 5 + carry 1 = 10. Write 0 and carry 1.

23. Column 3 from the right: 7 + 6 + carry 1 = 14. Write 4 and carry 1.

24. Column 4 from the right: 5 + 8 + carry 1 = 14. Write 4 and carry 1.

25. False track of the first addition: B=9, C=6, D=4, E=2, F=3, G=1, J=5. It does not extend into any substitution that satisfies the second addition; keeping only an exact first sum would therefore be insufficient.

26. The solver lists all the deductions and all the injection assignments compatible with the first addition, then tests the second without approximation. Only one complete substitution remains.

Common pitfalls

- Attribute a number to a letter forgetting that it is already taken.

- Allow an initial zero.

- Forget to postpone a deduction or change the value of a letter between the two additions.

Verification

Complete search by columns, binary deductions, rotation of unallocated numbers and arithmetic control of both equals.

Ten Letters, Two Locks

Cryptarithm / EXTREME

Answer

A = 7, B = 0, C = 1, D = 5, E = 4, F = 3, G = 8, H = 9, I = 6, J = 2

2660 + 3903 = 6563

1828 + 4766 = 6594

Reasoning

1. Each column carries a hold of 0 or 1. For an addition of two numbers to four digits, a possible fifth position of the result is a final hold, so is 1.

2. Write a column equation, from right to left: first-term digit + second-time digit + incoming hold = result digit + 10 × outgoing hold. The same letter must keep its value in both additions.

3. Addition 1, column 1: B + F + r1_0 = F + 10 × r1_1.

4. Addition 1, column 2: I + B + r1_1 = I + 10 × r1_2.

5. Addition 1, column 3: I + H + r1_2 = D + 10 × r1_3.

6. Addition 1, column 4: J + F + r1_3 = I + 10 × r1_4.

7. Addition 2, column 1: G + I + r2_0 = E + 10 × r2_1.

8. Addition 2, column 2: J + I + r2_1 = H + 10 × r2_2.

9. Addition 2, column 3: G + A + r2_2 = D + 10 × r2_3.

10. Addition 2, column 4: C + E + r2_3 = I + 10 × r2_4.

11. In these equations, r1_0 and r2_0 are zero. Each other retention is 0 or 1; the same letter always refers to the same number.

12. Explore possible pairs of digits in the column of units, calculate the restraint and bar any branch giving the same number to two different letters. Repeat for tens, hundreds and thousands; a letter already affected no longer receives any new choice.

13. The first addition admits 102 partial assignments respecting these constraints. After assigning letters still absent to free numbers, 2448 substitutions are controlled against the second addition.

14. Crossing the two additions is essential: the entire substitution must be transported, not separately resolve each line and then juxtapose two incompatible responses.

15. Addition 1: JIIB + FHBF = IDIF, i.e. 2660 + 3903 = 6563.

16. Column 1 from the right: 0 + 3 + carry 0 = 3. Write 3 and carry 0.

17. Column 2 from the right: 6 + 0 + carry 0 = 6. Write 6 and carry 0.

18. Column 3 from the right: 6 + 9 + carry 0 = 15. Write 5 and carry 1.

19. Column 4 from the right: 2 + 3 + carry 1 = 6. Write 6 and carry 0.

20. Addition 2: CGJG + EAII = IDHE, or 1828 + 4766 = 6594.

21. Column 1 from the right: 8 + 6 + carry 0 = 14. Write 4 and carry 1.

22. Column 2 from the right: 2 + 6 + carry 1 = 9. Write 9 and carry 0.

23. Column 3 from the right: 8 + 7 + carry 0 = 15. Write 5 and carry 1.

24. Column 4 from the right: 1 + 4 + carry 1 = 6. Write 6 and carry 0.

25. False track of the first addition: B=0, D=7, F=1, H=4, I=3, J=2. It does not extend into any substitution that satisfies the second addition; keeping only an exact first sum would therefore be insufficient.

26. The solver lists all the deductions and all the injection assignments compatible with the first addition, then tests the second without approximation. Only one complete substitution remains.

Common pitfalls

- Attribute a number to a letter forgetting that it is already taken.

- Allow an initial zero.

- Forget to postpone a deduction or change the value of a letter between the two additions.

Verification

Complete search by columns, binary deductions, rotation of unallocated numbers and arithmetic control of both equals.

The Seven Staircases

Futoshiki / EXPERT

Answer

Grid, line by line:

3 6 4 2 7 5 1

6 2 1 7 4 3 5

7 4 3 1 5 2 6

5 3 7 6 2 1 4

4 1 6 5 3 7 2

1 5 2 3 6 4 7

2 7 5 4 1 6 3

	1	2	3	4	5	6	7
1	3	6	4	2	7	5	1
2	6	2	1	7	4	3	5
3	7	4	3	1	5	2	6
4	5	3	7	6	2	1	4
5	4	1	6	5	3	7	2
6	1	5	2	3	6	4	7
7	2	7	5	4	1	6	3

Reasoning

1. First enter the candidates from 1 to 7 in each empty box. Remove any numbers already fixed in its line or column. The rows and columns each require a complete permutation.

2. Owns inequalities in both directions. If A < B, A can no longer take a value greater than or equal to the greatest candidate of B, and B can no longer take a value less than or equal to the smallest candidate of A.

3. After each reduction, check the unique locations of each digit in each row and column. A growing chain of k boxes consumes at least k separate values; its ends also give bounds.

4. The reference method combines these reductions and then controlled assumptions. On this problem, the exact search visits 13 knots, encounters 6 contradictions and reaches a depth of 3 nested choices. These numbers describe this solver and not a guaranteed human duration.

5. A hypothesis is treated as a temporary branch: notes its box and its number, spreads all the constraints, then returns to the last choice if a box loses all its candidates or if a line can no longer contain a number. Do not erase a hypothesis without noting the contradiction.

6. Here are the first assignments of the path that leads to the solution. "Assumption" means a choice to be confirmed in exploration, not a proven local deduction:

7. L3C6 = 2 - single location.

8. L7C5 = 1 - single location.

9. L3C4 = 1 - single location.

10. L2C6 = 3 - hypothesis.

11. L2C7 = 5 - inequality.

12. L3C7 = 6 - inequality.

13. L2C4 = 7 - row/column.

14. L7C4 = 4 - row/column.

15. L6C4 = 3 - inequality.

16. L5C4 = 5 - single location.

17. L1C7 = 1 - hypothesis.

18. L2C2 = 2 - hypothesis.

19. L2C3 = 1 - row/column.

20. L7C2 = 7 - row/column.

21. L4C6 = 1 - single location.

22. L4C3 = 7 - single location.

23. L7C1 = 2 - row/column.

24. L3C2 = 4 - row/column.

25. L6C6 = 4 - row/column.

26. L5C6 = 7 - single location.

27. L6C1 = 1 - inequality.

28. L6C5 = 6 - inequality.

29. L4C2 = 3 - inequality.

30. L3C1 = 7 - row/column.

31. L4C5 = 2 - row/column.

32. L4C1 = 5 - single location.

33. L6C2 = 5 - row/column.

34. L6C3 = 2 - row/column.

35. L1C2 = 6 - row/column.

36. L5C2 = 1 - row/column.

37. Example of a rebutted branch: supposes L2C6 = 1, then L1C5 = 3. The propagation results in: The number 5 can no longer be placed in the group L6C1, L6C2, L6C3, L6C4, L6C5, L6C6, L6C7 The certificate of exploration gives all branches, with indices of boxes counted from zero.

38. Example of a rebutted branch: supposes L2C6 = 1, then L1C5 = 7, then L2C3 = 2. The propagation results in: Two identical fixed digits in boxes L6C1, L6C2, L6C3, L6C4, L6C5, L6C6, L6C7 The certificate of exploration gives all branches, with indices of boxes counted from zero.

39. Example of a rebutted branch: supposes L2C6 = 1, then L1C5 = 7, then L2C3 = 7. The propagation results in: L-inequality L6C4 < L7C4 no longer has compatible torque values. The exploration certificate gives all branches, with indices of cells counted from zero.

40. The final grid allows line-by-line and column-by-column control: each one contains once all the numbers. Then check the inequalities one by one:

41. L5C1 < L4C1 : 4 < 5.

42. L6C6 < L5C6 : 4 < 7.

43. L3C6 < L3C7 : 2 < 6.

44. L2C6 < L2C7 : 3 < 5.

45. L6C2 < L7C2 : 5 < 7.

46. L4C1 < L3C1 : 5 < 7.

47. L7C7 < L7C6 : 3 < 6.

48. L5C7 < L6C7 : 2 < 7.

49. L5C2 < L4C2 : 1 < 3.

50. L1C4 < L1C5 : 2 < 7.

51. L1C4 < L2C4 : 2 < 7.

52. L6C1 < L7C1 : 1 < 2.

53. L6C4 < L7C4 : 3 < 4.

54. L6C6 < L6C5 : 4 < 6.

55. L4C2 < L3C2 : 3 < 4.

56. L4C5 < L3C5 : 2 < 5.

57. L2C7 < L3C7 : 5 < 6.

58. The exact counting continued after finding this grid to look for a second solution. All other branches fail: the solution is unique.

Common pitfalls

- Add the Sudoku regions: they are not part of Futoshiki.

- Forget that inequality forces its two ends.

- Take a search hypothesis for a figure deducted unconditionally.

Verification

Comprehensive research with different domains, constraints, unique locations and inequalities; independent control of the final grid.

The Eight Passages

Futoshiki / EXTREME

Answer

Grid, line by line:

1 5 6 8 2 7 4 3

8 6 2 3 4 5 1 7

6 8 3 2 7 1 5 4

4 7 5 1 6 3 2 8

3 2 4 7 1 6 8 5

2 3 7 4 5 8 6 1

7 4 1 5 8 2 3 6

5 1 8 6 3 4 7 2

	1	2	3	4	5	6	7	8
1	1	5	6	8	2	7	4	3
2	8	6	2	3	4	5	1	7
3	6	8	3	2	7	1	5	4
4	4	7	5	1	6	3	2	8
5	3	2	4	7	1	6	8	5
6	2	3	7	4	5	8	6	1
7	7	4	1	5	8	2	3	6
8	5	1	8	6	3	4	7	2

Reasoning

1. First enter the candidates from 1 to 8 in each empty box. Remove any numbers already fixed in its line or column. The rows and columns each require a complete permutation.

2. Owns inequalities in both directions. If A < B, A can no longer take a value greater than or equal to the greatest candidate of B, and B can no longer take a value less than or equal to the smallest candidate of A.

3. After each reduction, check the unique locations of each digit in each row and column. A growing chain of k boxes consumes at least k separate values; its ends also give bounds.

4. The reference method combines these reductions and then controlled assumptions. On this problem, the exact search visits 43 knots, encounters 21 contradictions and reaches a depth of 9 nested choices. These numbers describe this solver and not a guaranteed human duration.

5. A hypothesis is treated as a temporary branch: notes its box and its number, spreads all the constraints, then returns to the last choice if a box loses all its candidates or if a line can no longer contain a number. Do not erase a hypothesis without noting the contradiction.

6. Here are the first assignments of the path that leads to the solution. "Assumption" means a choice to be confirmed in exploration, not a proven local deduction:

7. L5C4 = 7 - inequality.

8. L5C1 = 3 - inequality.

9. L6C1 = 2 - inequality.

10. L6C4 = 4 - row/column.

11. L1C4 = 8 - row/column.

12. L1C3 = 6 - row/column.

13. L4C8 = 8 - single location.

14. L1C1 = 1 - hypothesis.

15. L1C7 = 4 - hypothesis.

16. L1C8 = 3 - row/column.

17. L3C2 = 8 - hypothesis.

18. L8C3 = 8 - single location.

19. L2C1 = 8 - single location.

20. L2C2 = 6 - hypothesis.

21. L4C2 = 7 - row/column.

22. L6C2 = 3 - row/column.

23. L7C2 = 4 - row/column.

24. L7C1 = 7 - row/column.

25. L7C8 = 6 - row/column.

26. L8C7 = 7 - single location.

27. L2C7 = 1 - hypothesis.

28. L7C7 = 3 - row/column.

29. L7C3 = 1 - row/column.

30. L2C6 = 5 - hypothesis.

31. L2C5 = 4 - hypothesis.

32. L2C8 = 7 - row/column.

33. L3C4 = 2 - hypothesis.

34. L3C7 = 5 - row/column.

35. L3C6 = 1 - only location.

36. L3C3 = 3 - single location.

37. Example of a rebutted branch: supposes L1C1 = 1, then L1C7 = 3, then L3C2 = 7. The propagation results in: The box L3C4 no longer has any candidate after excluding the numbers set in its line/column. The exploration certificate gives all branches, with indices of boxes counted from zero.

38. Example of a rebutted branch: supposes L1C1 = 1, then L1C7 = 3, then L3C2 = 8, then L2C2 = 6, then L2C8 = 5. The propagation results in: The box L3C5 no longer has any candidate after excluding the numbers set in its line/column. The certificate of exploration gives all branches, with indices of cells counted from scratch.

39. Example of a rebutted branch: supposes L1C1 = 1, then L1C7 = 3, then L3C2 = 8, then L2C2 = 6, then L2C8 = 7, then L3C4 = 1. The propagation results in: The number 2 can no longer be placed in group L1C7, L2C7, L3C7, L4C7, L5C7, L6C7, L7C7, L8C7 The certificate of exploration gives all branches, with indices of cells counted from scratch.

40. The final grid allows line-by-line and column-by-column control: each one contains once all the numbers. Then check the inequalities one by one:

41. L4C1 < L4C2 : 4 < 7.

42. L5C6 < L5C7 : 6 < 8.

43. L8C2 < L7C2 : 1 < 4.

44. L1C5 < L1C4 : 2 < 8.

45. L6C2 < L6C3 : 3 < 7.

46. L1C2 < L2C2 : 5 < 6.

47. L5C1 < L4C1 : 3 < 4.

48. L4C5 < L3C5 : 6 < 7.

49. L2C3 < L2C2 : 2 < 6.

50. L4C3 < L4C2 : 5 < 7.

51. L6C1 < L6C2 : 2 < 3.

52. L3C6 < L3C7 : 1 < 5.

53. L7C3 < L6C3 : 1 < 7.

54. L6C4 < L5C4 : 4 < 7.

55. L8C8 < L8C7 : 2 < 7.

56. L5C2 < L4C2 : 2 < 7.

57. L4C2 < L3C2 : 7 < 8.

58. L1C2 < L1C3 : 5 < 6.

59. L6C1 < L5C1 : 2 < 3.

60. The exact counting continued after finding this grid to look for a second solution. All other branches fail: the solution is unique.

Common pitfalls

- Add the Sudoku regions: they are not part of Futoshiki.

- Forget that inequality forces its two ends.

- Take a search hypothesis for a figure deducted unconditionally.

Verification

Comprehensive research with different domains, constraints, unique locations and inequalities; independent control of the final grid.

Columns Under Tension

Futoshiki / EXTREME

Answer

Grid, line by line:

4 8 6 3 5 2 1 7

7 1 2 5 8 4 6 3

6 3 8 4 7 1 5 2

2 5 1 7 3 6 8 4

8 4 3 6 2 5 7 1

5 2 7 1 6 3 4 8

1 7 5 2 4 8 3 6

3 6 4 8 1 7 2 5

	1	2	3	4	5	6	7	8
1	4	8	6	3	5	2	1	7
2	7	1	2	5	8	4	6	3
3	6	3	8	4	7	1	5	2
4	2	5	1	7	3	6	8	4
5	8	4	3	6	2	5	7	1
6	5	2	7	1	6	3	4	8
7	1	7	5	2	4	8	3	6
8	3	6	4	8	1	7	2	5

Reasoning

1. First enter the candidates from 1 to 8 in each empty box. Remove any numbers already fixed in its line or column. The rows and columns each require a complete permutation.

2. Owns inequalities in both directions. If A < B, A can no longer take a value greater than or equal to the greatest candidate of B, and B can no longer take a value less than or equal to the smallest candidate of A.

3. After each reduction, check the unique locations of each digit in each row and column. A growing chain of k boxes consumes at least k separate values; its ends also give bounds.

4. The reference method combines these reductions and then controlled assumptions. On this problem, the exact search visits 55 knots, encounters 27 contradictions and reaches a depth of 9 nested choices. These numbers describe this solver and not a guaranteed human duration.

5. A hypothesis is treated as a temporary branch: notes its box and its number, spreads all the constraints, then returns to the last choice if a box loses all its candidates or if a line can no longer contain a number. Do not erase a hypothesis without noting the contradiction.

6. Here are the first assignments of the path that leads to the solution. "Assumption" means a choice to be confirmed in exploration, not a proven local deduction:

7. L4C5 = 3 - inequality.

8. L4C4 = 7 - inequality.

9. L3C6 = 1 - inequality.

10. L5C6 = 5 - inequality.

11. L4C6 = 6 - row/column.

12. L8C4 = 8 - single location.

13. L1C4 = 3 - hypothesis.

14. L3C5 = 7 - hypothesis.

15. L2C5 = 8 - hypothesis.

16. L7C6 = 8 - single location.

17. L1C5 = 5 - hypothesis.

18. L7C5 = 4 - row/column.

19. L2C6 = 4 - hypothesis.

20. L8C6 = 7 - row/column.

21. L7C3 = 5 - single location.

22. L6C1 = 5 - single location.

23. L4C1 = 2 - hypothesis.

24. L4C3 = 1 - row/column.

25. L8C2 = 6 - inequality.

26. L2C1 = 7 - hypothesis.

27. L2C7 = 6 - single location.

28. L2C3 = 2 - row/column.

29. L8C3 = 4 - row/column.

30. L2C2 = 1 - inequality.

31. L2C8 = 3 - row/column.

32. L8C1 = 3 - row/column.

33. L8C7 = 2 - single location.

34. L3C1 = 6 - row/column.

35. L7C1 = 1 - row/column.

36. L1C1 = 4 - single location.

37. Example of a rebutted branch: supposes L1C4 = 1. The propagation results in: Two identical fixed digits in boxes L3C1, L3C2, L3C3, L3C4, L3C5, L3C6, L3C7, L3C8 The certificate of exploration gives all branches, with indices of squares counted from zero.

38. Example of a rebutted branch: supposes L1C4 = 3, then L3C5 = 7, then L2C5 = 4, then L1C8 = 1. The propagation results in: L-inequality L8C3 < L7C3 no longer has compatible torque values. The exploration certificate gives all branches, with indices of squares counted from zero.

39. Example of a rebutted branch: supposes L1C4 = 3, then L3C5 = 7, then L2C5 = 4, then L1C8 = 7. Propagation results in: The number 4 can no longer be placed in the group L7C1, L7C2, L7C3, L7C4, L7C5, L7C6, L7C7, L7C8 The certificate of exploration gives all branches, with indices of boxes counted from zero.

40. The final grid allows line-by-line and column-by-column control: each one contains once all the numbers. Then check the inequalities one by one:

41. L4C5 < L4C4 : 3 < 7.

42. L8C6 < L7C6 : 7 < 8.

43. L1C4 < L1C3 : 3 < 6.

44. L2C3 < L1C3 : 2 < 6.

45. L2C2 < L2C3 : 1 < 2.

46. L3C6 < L3C7 : 1 < 5.

47. L1C7 < L2C7 : 1 < 6.

48. L6C6 < L6C7 : 3 < 4.

49. L6C6 < L5C6 : 3 < 5.

50. L8C1 < L8C2 : 3 < 6.

51. L8C3 < L7C3 : 4 < 5.

52. L7C4 < L7C5 : 2 < 4.

53. L3C1 < L2C1 : 6 < 7.

54. L6C2 < L5C2 : 2 < 4.

55. L6C4 < L6C5 : 1 < 6.

56. L5C6 < L4C6 : 5 < 6.

57. L6C2 < L7C2 : 2 < 7.

58. L7C1 < L8C1 : 1 < 3.

59. L7C5 < L7C6 : 4 < 8.

60. The exact counting continued after finding this grid to look for a second solution. All other branches fail: the solution is unique.

Common pitfalls

- Add the Sudoku regions: they are not part of Futoshiki.

- Forget that inequality forces its two ends.

- Take a search hypothesis for a figure deducted unconditionally.

Verification

Comprehensive research with different domains, constraints, unique locations and inequalities; independent control of the final grid.

Crossing Intervals

Nonogram / EXPERT

Answer

The grid below is the only solution; ■ = black, · = empty.

	1	2	3	4	5	6	7	8	9	10	11	12
1	■	·	■	■	■	■	■	·	·	■	·	·
2	·	■	·	■	■	·	·	·	■	·	■	■
3	■	·	·	·	■	·	■	·	·	·	·	·
4	■	■	■	·	·	■	·	■	·	■	■	■
5	■	■	■	·	·	■	■	·	■	·	■	■
6	·	·	·	■	·	■	·	·	·	·	■	·
7	·	■	■	■	·	·	·	■	·	■	·	■
8	·	·	·	·	■	·	■	■	■	·	·	·
9	■	·	■	■	■	■	■	·	·	·	·	·
10	■	·	·	■	■	·	■	·	■	·	·	■
11	■	■	·	■	·	■	·	·	·	■	■	·
12	·	■	■	·	·	■	·	■	■	·	·	■

Reasoning

1. Start by listing the investments in each series. Keep only those compatible with the already fixed crosses: any box common to all investments can be fixed.

2. This intersecting closure is not enough here. Case analyses should be conducted on a line or column, each choice should be passed on to the perpendicular lines and a choice should be rejected as soon as a series can no longer be placed.

3. The exhaustive control visited 9 states, encountered 6 contradictions and reached 2 levels of assumptions. These measures describe this solver; they do not represent a universal ranking of players.

4. L1: 20 compatible placements; L1C6 black, L1C7 black.

5. L4: 5 compatible placements; Black L4C2, Black L4C3, Black L4C10, Black L4C11.

6. L5: 5 compatible placements; L5C2 black, L5C3 black, L5C6 black, L5C11 black.

7. C2: 12 compatible placements; L3C2 empty, L6C2 empty.

8. C3: 8 compatible placements; L3C3 empty, L6C3 empty.

9. C6: 10 compatible placements. L2C6 is empty.
10. C7: 20 compatible placements. L2C7 is empty.
11. C10: 40 compatible placements; L3C10 empty, L5C10 empty.
12. L2: 9 compatible placements; L2C4 black, L2C11 black.
13. L5: 4 compatible placements. L5C12 is filled.
14. C11: 5 compatible placements; L1C11 empty, L3C11 empty, L6C11 black, L7C11 empty.
15. L6: 16 compatible investments; L6C10 empty, L6C12 empty.
16. C12: 8 compatible placements; L3C12 empty, black L4C12.
17. L4: 4 compatible placements. L4C9 is empty.
18. Case analysis on L4 : placement 3/4 = ■^^··^^^^^^.
19. C4: 12 compatible placements. L10C4 is filled.
20. Case analysis on L5 : placement 3/4 = ■^^^^^^^^^^.
21. C4: 6 compatible placements; L7C4 black, L11C4 black.
22. C7: 7 compatible placements. L6C7 is empty.
23. C8: 17 compatible placements. L2C8 is empty.
24. C9: 22 compatible placements. L6C9 is empty.
25. L2: 3 compatible placements; L2C9 black, L2C10 empty, L2C12 black.
26. L7: 11 compatible placements. L7C1 is empty.
27. C1: 9 compatible placements. L10C1 is filled.
28. C9: 4 compatible placements; L1C9 empty, L3C9 empty.
29. C12: 4 compatible placements. L1C12 is empty.
30. L1: 3 compatible placements; L1C4 black, L1C5 black, L1C10 black.
31. L10: 14 compatible placements. L10C2 is empty.
32. C2: 6 compatible placements; black L11C2, black L12C2.
33. C4: 3 compatible placements. L3C4 is empty.
34. C5: 5 compatible placements; L2C5 black, L3C5 black.
35. L2: 2 compatible placements. L2C3 is empty.
36. L3: 2 compatible placements; L3C1 black, L3C6 empty.
37. L11: 10 compatible placements; L11C1 black, L11C3 empty, L11C5 empty.
38. C1: 2 compatible placements; black L1C1, empty L2C1, empty L6C1, empty L8C1.
39. C3: 3 compatible placements; L1C3 black, L12C3 black.
40. C5: 3 compatible placements; L8C5 black, L12C5 empty.
41. C6: 3 compatible placements; black L6C6, empty L7C6, black L11C6.

42. L1: 1 compatible placements; L1C2 empty, L1C8 empty.

43. L2: 1 compatible placements. L2C2 is filled.

44. L6: 2 compatible placements. L6C5 is empty.

45. L7: 6 compatible placements; L7C3 black, L7C12 black.

46. L11: 4 compatible placements. L11C7 is empty.

47. L12: 4 compatible placements; L12C1 empty, L12C4 empty, L12C9 black.

48. C1: 1 compatible placements. L9C1 is filled.

49. C3: 2 compatible placements. L8C3 is empty.

50. C4: 1 compatible placements; L6C4 black, L8C4 empty, L9C4 black.

51. C5: 2 compatible placements. L9C5 is filled.

52. C7: 2 compatible placements; L3C7 black, L8C7 black, L9C7 black, L12C7 empty.

53. C8: 10 compatible placements. L3C8 is empty.

54. C9: 3 compatible placements. L11C9 is empty.

55. C12: 3 compatible placements. L8C12 is empty.

56. L6: 1 compatible placements. L6C8 is empty.

57. L8: 2 compatible placements; L8C10 empty, L8C11 empty.

58. L9: 2 compatible placements; L9C2 empty, L9C6 black, L9C9 empty, L9C10 empty, L9C11 empty, L9C12 empty.

59. L11: 2 compatible placements; L11C8 empty, L11C11 black.

60. L12: 3 compatible placements. L12C6 is filled.

61. C6: 1 compatible placements; L8C6 empty, L10C6 empty.

62. C9: 2 compatible placements. L10C9 is filled.

63. C11: 1 compatible placements; L10C11 empty, L12C11 empty.

64. C12: 1 compatible placements; L10C12 black, L11C12 empty, L12C12 black.

65. L8: 1 compatible placements; L8C2 empty, L8C8 black, L8C9 black.

66. L10: 2 compatible placements; L10C7 black, L10C8 empty, L10C10 empty.

67. L11: 1 compatible placements. L11C10 is filled.

68. C2: 1 compatible placements. L7C2 is filled.

69. C7: 1 compatible placements. L7C7 is empty.

70. C8: 2 compatible placements. L12C8 is filled.

71. C9: 1 compatible placements. L7C9 is empty.

72. C10: 1 compatible placements; L7C10 black, L12C10 empty.

73. L7: 1 compatible placements; L7C5 empty, L7C8 black.

74. C5: 1 compatible placements. L10C5 is filled.

75. C8: 1 compatible placements. L9C8 is empty.

76. L9: 1 compatible placements. L9C3 is filled.

77. L10: 1 compatible placements. L10C3 is empty.

78. Examples of contradictions encountered:

79. C7 has no legal placement left at depth 1.

80. L2 has no legal placement left at depth 1.

81. C5 has no legal placement left at depth 2.

82. C5 has no legal placement left at depth 2.

83. C3 has no legal placement left at depth 2.

84. L12 has no legal placement left at depth 1.

Common pitfalls

- Do not confuse an unknown box with an empty proven box.

- A hypothesis must be cancelled with all its consequences if a series becomes impossible.

- The form suggested by the first black boxes is not evidence.

Verification

Enumeration of placements by line, propagation by intersections and exhaustive search for up to two solutions; new verification of response series.

The Suspended Weave

Nonogram / EXPERT

Answer

The grid below is the only solution; ■ = black, · = empty.

	1	2	3	4	5	6	7	8	9	10	11	12
1	■	■	·	·	·	■	■	·	■	·	·	■
2	·	·	■	·	·	■	·	·	■	·	·	■
3	·	■	■	■	■	·	·	·	·	■	·	■
4	■	■	·	·	·	■	■	■	·	■	■	■
5	■	■	·	■	■	·	■	■	·	·	·	·
6	■	■	■	·	·	■	·	·	■	■	·	·
7	■	■	■	■	·	·	■	·	■	·	·	·
8	■	■	■	■	·	·	·	·	·	■	·	■
9	·	■	■	■	■	·	■	·	·	·	■	■
10	·	·	·	·	·	·	■	·	·	■	■	·
11	■	■	■	■	■	·	■	■	·	·	■	·
12	■	■	■	·	·	·	·	■	·	■	·	·

Reasoning

1. Start by listing the investments in each series. Keep only those compatible with the already fixed crosses: any box common to all investments can be fixed.

2. This intersecting closure is not enough here. Case analyses should be conducted on a line or column, each choice should be passed on to the perpendicular lines and a choice should be rejected as soon as a series can no longer be placed.

3. The exhaustive control visited 14 states, encountered 8 contradictions and reached 5 levels of assumptions. These measures describe this solver; they do not represent a universal ranking of players.

4. L4: 10 compatible placements; L4C6 black, L4C10 black.

5. L9: 20 compatible placements. L9C4 is filled.

6. L11: 10 compatible placements; Black L11C3, Black L11C4, Black L11C5.

7. C1: 10 compatible placements; black L5C1, black L6C1, black L7C1.

8. C2: 1 compatible placements; black L1C2, empty L2C2, black L3C2, black L4C2, black L5C2, black L6C2, black L7C2, black L8C2, black L9C2, black L10C2, empty L11C2, black L12C2.

9. C3: 9 compatible placements; L6C3 black, L7C3 black.

10. C4: 6 compatible placements; L6C4 empty, L7C4 black, L8C4 black, L10C4 empty, L12C4 empty.

11. C5: 35 compatible placements; L10C5 empty, L12C5 empty.

12. C7: 15 compatible placements. L10C7 is filled.

13. C10: 11 compatible placements. L5C10 is empty.

14. L3: 25 compatible placements; L3C3 black, L3C4 black.

15. L5: 8 compatible placements. L5C3 is empty.

16. L7: 15 compatible placements. L7C5 is empty.

17. L8: 25 compatible placements. L8C3 is filled.

18. L9: 16 compatible placements. L9C3 is filled.

19. L12: 15 compatible placements; black L12C1, black L12C3.

20. C1: 6 compatible placements; L10C1 empty, L11C1 black.

21. C3: 2 compatible placements; L1C3 empty, L10C3 empty.

22. C4: 2 compatible placements; L2C4 empty, L4C4 empty.

23. L1: 20 compatible placements. L1C1 is filled.

24. L4: 6 compatible placements; L4C7 black, L4C11 black.

25. L10: 3 compatible placements; L10C6 empty, L10C8 empty.

26. L11: 6 compatible placements. L11C6 is empty.

27. C1: 3 compatible placements. L2C1 is empty.

28. Case analysis on C3 : placement 1/2 = * * * * * * * * * * *

29. L4: 3 compatible placements. L4C1 is filled.

30. C1: 2 compatible placements. L9C1 is empty.

31. L9: 6 compatible placements; L9C5 black, L9C6 empty.

32. C5: 10 compatible placements. L8C5 is empty.

33. L8: 15 compatible placements. L8C1 is filled.

34. C1: 1 compatible placements. L3C1 is empty.

35. L3: 10 compatible placements; L3C5 black, L3C6 empty.

36. C5: 3 compatible placements; L2C5 empty, L4C5 empty.

37. L4: 1 compatible placements; L4C8 black, L4C9 empty, L4C12 black.

38. Case analysis on C4 : placement 2/2 = * * * * * * * * * * *

39. L5: 3 compatible placements; L5C5 black, L5C6 empty.

40. C5: 1 compatible investments; L1C5 empty, L6C5 empty.

41. C6: 4 compatible placements; L1C6 black, L2C6 black.

42. L1: 3 compatible placements; L1C7 black, L1C8 empty.

43. L2: 6 compatible placements. L2C7 is empty.

44. C8: 7 compatible placements. L2C8 is empty.

45. Case analysis on L1: placement 2/3 = ■·····

46. C9: 7 compatible placements; L2C9 black, L3C9 empty.

47. C10: 1 compatible placements; L2C10 empty, L3C10 black, L6C10 black, L7C10 empty, L8C10 black, L9C10 empty, L10C10 black, L11C10 empty, L12C10 black.

48. C11: 6 compatible placements. L3C11 is empty.

49. C12: 6 compatible placements; L2C12 black, L3C12 black, L5C12 empty.

50. L2: 1 compatible placements. L2C11 is empty.

51. L3: 1 compatible placements; L3C7 empty, L3C8 empty.

52. L5: 2 compatible placements; L5C8 black, L5C11 empty.

53. L6: 5 compatible placements. L6C12 is empty.

54. L8: 4 compatible placements; L8C9 empty, L8C11 empty.

55. L9: 3 compatible placements; L9C11 black, L9C12 black.

56. L10: 2 compatible placements. L10C12 is empty.

57. L11: 4 compatible placements. L11C8 is filled.

58. L12: 4 compatible placements; L12C9 empty, L12C11 empty.

59. C7: 3 compatible placements; L5C7 black, L6C7 empty, L11C7 black.

60. C8: 1 compatible placements; L6C8 empty, L7C8 empty, L8C8 empty, L9C8 empty, L12C8 black.

61. C11: 1 compatible placements; L6C11 empty, L7C11 empty, L10C11 black, L11C11 black.

62. C12: 1 compatible placements; L7C12 empty, L8C12 black, L11C12 empty, L12C12 empty.

63. L5: 1 compatible placements. L5C9 is empty.

64. L6: 1 compatible placements; L6C6 black, L6C9 black.

65. L7: 2 compatible placements. L7C9 is filled.

66. L8: 1 compatible placements; L8C6 empty, L8C7 empty.

67. L10: 1 compatible placements. L10C9 is empty.

68. L11: 1 compatible placements. L11C9 is empty.

69. L12: 1 compatible placements; L12C6 empty, L12C7 empty.

70. C6: 1 compatible placements. L7C6 is empty.

71. C7: 1 compatible placements; L7C7 black, L9C7 black.

72. C9: 1 compatible placements. L9C9 is empty.

73. Examples of contradictions encountered:

74. C10 has no legal placement left at depth 2.

75. L9 has no legal placement left at depth 3.

76. C10 has no legal placement left at depth 4.

77. C12 has no legal placement left at depth 5.

78. L7 has no legal placement left at depth 5.

79. C7 has no legal placement left at depth 5.

80. L7 has no legal placement left at depth 4.

81. L1 has no legal placement left at depth 1.

Common pitfalls

- Do not confuse an unknown box with an empty proven box.

- A hypothesis must be cancelled with all its consequences if a series becomes impossible.

- The form suggested by the first black boxes is not evidence.

Verification

Enumeration of placements by line, propagation by intersections and exhaustive search for up to two solutions; new verification of response series.

The Fragmented Grid

Nonogram / EXTREME

Answer

The grid below is the only solution; ■ = black, · = empty.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	·	■	·	·	·	·	■	·	·	·	·	■	■	·	·	·
2	■	·	■	■	·	■	■	·	·	·	■	·	■	·	■	·
3	·	■	■	·	·	·	·	·	·	·	·	■	·	■	■	·
4	■	·	·	·	·	■	·	■	■	·	·	■	■	·	■	■
5	·	■	■	·	■	·	·	·	·	■	·	■	·	·	■	■
6	·	·	·	·	■	·	·	·	■	·	■	■	■	·	·	■
7	■	■	■	·	·	■	■	·	·	■	■	■	·	■	■	■
8	·	·	·	■	■	·	·	·	·	·	■	■	·	·	■	·
9	·	·	■	·	■	■	■	·	■	·	■	■	■	■	·	·
10	·	■	·	·	·	■	·	·	■	■	■	·	■	·	■	■
11	·	·	·	·	■	·	·	■	·	·	■	·	■	■	·	·
12	·	■	·	·	■	·	·	■	■	■	·	·	·	■	■	·
13	■	·	·	·	·	■	■	·	■	■	·	■	·	■	■	■
14	·	■	·	·	■	·	■	·	·	■	·	■	·	■	■	·
15	■	·	·	·	·	·	·	■	■	·	■	■	·	■	·	■
16	■	■	■	·	■	·	·	·	■	■	■	·	■	■	■	·

Reasoning

1. Start by listing the investments in each series. Keep only those compatible with the already fixed crosses: any box common to all investments can be fixed.

2. This intersecting closure is not enough here. Case analyses should be conducted on a line or column, each choice should be passed on to the perpendicular lines and a choice should be rejected as soon as a series can no longer be placed.

3. The exhaustive control visited 52 states, encountered 34 contradictions and reached 7 levels of assumptions. These measures describe this solver; they do not represent a universal ranking of players.

4. L7: 15 compatible placements; L7C3 black, L7C10 black, L7C14 black.

5. L10: 35 compatible placements; L10C1 empty, L10C3 empty.

6. L13: 31 compatible placements. L13C14 is filled.

7. C2: 5 compatible placements; L9C2 empty, L11C2 empty, L12C2 black, L13C2 empty, L14C2 black, L15C2 empty, L16C2 black.

8. C3: 74 compatible placements. L8C3 is empty.

9. C10: 62 compatible placements. L6C10 is empty.

10. C11: 56 compatible placements. L8C11 is filled.

11. C12: 20 compatible placements; L6C12 black, L7C12 black, L8C12 black, L9C12 black.

12. C14: 23 compatible placements; L6C14 empty, L8C14 empty, L11C14 black, L12C14 black, L14C14 black.

13. C15: 6 compatible placements; L2C15 black, L3C15 black, L4C15 black, L7C15 black, L12C15 black, L13C15 black.

14. L2: 21 compatible placements; L2C14 empty, L2C16 empty.

15. L7: 6 compatible placements; L7C9 empty, L7C11 black, L7C13 empty, L7C16 black.

16. L8: 12 compatible placements; L8C10 empty, L8C13 empty.

17. L9: 25 compatible placements. L9C16 is empty.

18. L11: 155 compatible placements. L11C16 is empty.

19. L12: 15 compatible placements; L12C1 empty, L12C3 empty, L12C13 empty, L12C16 empty.

20. L14: 20 compatible placements; L14C1 empty, L14C3 empty, L14C16 empty.

21. L16: 30 compatible placements. L16C3 is filled.

22. C1: 113 compatible placements; L15C1 black, L16C1 black.

23. C3: 10 compatible placements; L6C3 empty, L15C3 empty.

24. C13: 4 compatible placements; L1C13 black, L2C13 black, L3C13 empty, L4C13 black, L5C13 empty, L6C13 black, L9C13 black, L10C13 black, L11C13 black.

25. C16: 4 compatible placements; L1C16 empty, L3C16 empty, L5C16 black, L6C16 black, L10C16 black, L13C16 black.

26. L1: 81 compatible placements. L1C15 is empty.

27. L2: 15 compatible placements. L2C12 is empty.

28. L3: 45 compatible placements. L3C14 is filled.

29. L4: 20 compatible placements; L4C11 empty, L4C12 black, L4C14 empty, L4C16 black.

30. L5: 70 compatible placements; L5C14 empty, L5C15 black.

31. L6: 22 compatible placements; L6C11 black, L6C15 empty.

32. L10: 10 compatible placements; L10C12 empty, L10C14 empty, L10C15 black.

33. L11: 63 compatible placements; L11C12 empty, L11C15 empty.

34. L13: 11 compatible placements. L13C13 is empty.

35. L16: 20 compatible placements. L16C4 is empty.

36. C11: 21 compatible placements; L9C11 black, L10C11 black.

37. C12: 3 compatible placements; L1C12 black, L3C12 black, L5C12 black, L14C12 black.

38. C14: 2 compatible placements; L15C14 black, L16C14 black.

39. C15: 1 compatible placements; L8C15 black, L9C15 empty, L14C15 black, L15C15 empty, L16C15 black.

40. C16: 2 compatible placements. L8C16 is empty.

41. L1: 36 compatible investments; L1C11 empty, L1C14 empty.

42. L3: 9 compatible placements. L3C11 is empty.

43. L5: 35 compatible placements. L5C11 is empty.

44. L9: 10 compatible placements. L9C5 is filled.

45. L10: 4 compatible placements; L10C8 empty, L10C9 black, L10C10 black.

46. L14: 10 compatible placements; L14C11 empty, L14C13 empty.

47. L15: 25 compatible placements. L15C13 is empty.

48. C9: 17 compatible placements. L8C9 is empty.

49. C11: 1 compatible placements; L2C11 black, L11C11 black, L12C11 empty, L13C11 empty, L15C11 black, L16C11 black.

50. C13: 1 compatible placements. L16C13 is filled.

51. C14: 1 compatible placements. L9C14 is filled.

52. L2: 10 compatible placements. L2C10 is empty.

53. L9: 7 compatible placements. L9C10 is empty.

54. L11: 22 compatible placements. L11C10 is empty.

55. L12: 6 compatible placements; L12C8 black, L12C12 empty.

56. L16: 3 compatible placements; L16C8 empty, L16C9 black, L16C10 black, L16C12 empty, L16C16 empty.

57. C9: 12 compatible placements; L14C9 empty, L15C9 black.

58. C10: 3 compatible placements; L12C10 black, L13C10 black, L14C10 black, L15C10 empty.

59. C12: 1 compatible placements; L13C12 black, L15C12 black.

60. C16: 1 compatible placements. L15C16 is filled.

61. L12: 3 compatible placements; L12C7 empty, L12C9 black.

62. L13: 4 compatible placements; L13C8 empty, L13C9 black.

63. L15: 1 compatible placements; L15C4 empty, L15C5 empty, L15C6 empty, L15C7 empty, L15C8 black.

64. C7: 71 compatible placements. L16C7 is empty.

65. C8: 9 compatible placements; L11C8 black, L14C8 empty.

66. C9: 10 compatible placements; L9C9 black, L11C9 empty.

67. L9: 4 compatible placements. L9C8 is empty.

68. L11: 5 compatible placements. L11C7 is empty.

69. Case analysis on L16: placement 1/2 = ■^^^^..^^^^^^^..

70. Case analysis on L12: placement 2/3 = * * * * * * * * * *

71. C5: 14 compatible placements. L13C5 is empty.

72. L13: 2 compatible placements; L13C6 black, L13C7 black.

73. C6: 35 compatible placements. L14C6 is empty.

74. C7: 35 compatible placements. L14C7 is filled.

75. Case analysis on L13: placement 1/2 =·····

76. Case analysis on L14: placement 2/2 = ■·····

77. C5: 5 compatible placements; L7C5 empty, L8C5 black, L10C5 empty, L11C5 black.

78. L7: 4 compatible placements; L7C2 black, L7C7 black.

79. L8: 2 compatible placements; L8C1 empty, L8C2 empty, L8C7 empty, L8C8 empty.

80. L11: 1 compatible placements; L11C1 empty, L11C3 empty, L11C4 empty, L11C6 empty.

81. C2: 1 compatible placements; black L1C2, empty L2C2, black L3C2, empty L4C2, black L5C2, empty L6C2.

82. C3: 3 compatible placements; L2C3 black, L9C3 black.

83. C7: 11 compatible placements. L6C7 is empty.

84. L1: 7 compatible placements; L1C1 empty, L1C3 empty.

85. L3: 2 compatible placements; L3C4 empty, L3C5 empty, L3C6 empty, L3C7 empty, L3C8 empty, L3C9 empty, L3C10 empty.

86. C3: 1 compatible placements; L3C3 black, L4C3 empty, L5C3 black.

87. C6: 8 compatible placements. L9C6 is filled.

88. L3: 1 compatible placements. L3C1 is empty.

89. L5: 10 compatible placements; L5C1 empty, L5C4 empty.

90. L9: 1 compatible placements; L9C1 empty, L9C4 empty, L9C7 black.

91. C1: 2 compatible placements; L2C1 black, L4C1 black.
92. C7: 2 compatible placements. L10C7 is empty.
93. L2: 3 compatible placements; L2C4 black, L2C5 empty.
94. C4: 5 compatible placements. L1C4 is empty.
95. C5: 2 compatible placements; L1C5 empty, L5C5 black.
96. L5: 4 compatible placements. L5C6 is empty.
97. C6: 6 compatible placements. L4C6 is filled.
98. L4: 2 compatible placements. L4C5 is empty.
99. C5: 1 compatible placements. L6C5 is filled.
100. L6: 3 compatible placements; L6C4 empty, L6C6 empty.
101. C6: 2 compatible placements; black L7C6, empty L8C6, black L10C6.
102. L7: 2 compatible placements. L7C8 is empty.
103. L8: 1 compatible placements. L8C4 is filled.
104. L10: 1 compatible placements. L10C4 is empty.
105. C4: 1 compatible placements; L4C4 empty, L7C4 empty.
106. L4: 1 compatible placements; L4C7 empty, L4C8 black, L4C9 black.
107. L7: 1 compatible placements. L7C1 is filled.
108. C1: 1 compatible placements. L6C1 is empty.
109. C7: 1 compatible placements; L1C7 black, L2C7 black, L5C7 empty.
110. C8: 1 compatible placements; L1C8 empty, L2C8 empty, L5C8 empty, L6C8 empty.
111. C9: 3 compatible placements. L5C9 is empty.
112. L1: 1 compatible placements; L1C6 empty, L1C9 empty, L1C10 empty.
113. L2: 1 compatible placements; L2C6 black, L2C9 empty.
114. L5: 1 compatible placements. L5C10 is filled.
115. L6: 1 compatible placements. L6C9 is filled.
116. Examples of contradictions encountered:
117. L7 has no legal placement left at depth 3.
118. C5 has no legal placement left at depth 4.
119. L2 has no legal placement left at depth 6.
120. C3 has no legal placement left at depth 6.
121. C5 has no legal placement left at depth 6.
122. L10 has no legal placement left at depth 7.
123. C5 has no legal placement left at depth 7.

124. C5 has no legal placement left at depth 7.

125. C1 has no legal placement left at depth 5.

126. L2 has no legal placement left at depth 6.

127. C3 has no legal placement left at depth 6.

128. C7 has no legal placement left at depth 6.

129. L10 has no legal placement left at depth 7.

130. C7 has no legal placement left at depth 7.

131. C8 has no legal placement left at depth 7.

132. C7 has no legal placement left at depth 4.

133. C3 has no legal placement left at depth 6.

134. C3 has no legal placement left at depth 6.

135. C5 has no legal placement left at depth 6.

136. L10 has no legal placement left at depth 7.

Common pitfalls

- Do not confuse an unknown box with an empty proven box.

- A hypothesis must be cancelled with all its consequences if a series becomes impossible.

- The form suggested by the first black boxes is not evidence.

Verification

Enumeration of placements by line, propagation by intersections and exhaustive search for up to two solutions; new verification of response series.

The Hidden Border

Nonogram / EXTREME

Answer

The grid below is the only solution; ■ = black, · = empty.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	■	·	■	■	■	·	■	■	■	·	■	·	·	·	·	■
2	·	·	■	■	■	■	■	■	■	■	■	·	·	·	■	·
3	·	■	■	·	·	·	■	·	■	·	■	·	■	■	■	·
4	·	■	■	·	■	·	■	·	■	·	■	■	■	·	■	■
5	■	■	·	·	■	■	·	■	·	·	·	■	·	·	■	■
6	·	■	■	■	■	■	·	·	■	■	■	·	■	■	·	·
7	■	■	·	■	■	■	·	·	■	■	·	·	■	■	·	■
8	■	·	·	■	■	·	■	■	■	■	■	·	·	·	■	■
9	■	·	■	■	·	·	·	■	·	■	·	■	·	·	·	·
10	■	·	■	·	·	■	·	·	■	■	·	·	·	■	■	■
11	·	■	■	·	·	·	·	·	·	■	■	·	■	·	·	·
12	·	·	·	·	·	■	·	■	·	■	·	·	■	·	■	■
13	·	■	·	■	■	·	■	·	■	■	·	·	■	·	·	·
14	·	■	·	■	·	■	·	·	·	■	·	■	■	■	·	·
15	·	·	·	·	·	·	·	■	·	·	·	·	■	■	■	·
16	·	·	·	·	·	■	·	·	·	■	■	■	·	·	·	■

Reasoning

1. Start by listing the investments in each series. Keep only those compatible with the already fixed crosses: any box common to all investments can be fixed.

2. This intersecting closure is not enough here. Case analyses should be conducted on a line or column, each choice should be passed on to the perpendicular lines and a choice should be rejected as soon as a series can no longer be placed.

3. The comprehensive control visited 29 states, encountered 16 contradictions and reached 10 levels of assumptions. These measures describe this solver; they do not represent a universal ranking of players.

4. L2: 21 compatible placements; L2C6 black, L2C7 black, L2C8 black, L2C9 black.

5. L4: 7 compatible placements; L4C2 black, L4C11 black, L4C12 black, L4C15 black.

6. L6: 23 compatible placements. L6C5 is filled.

7. L7: 21 compatible placements. L7C6 is filled.

8. L8: 12 compatible placements; L8C4 black, L8C7 black, L8C8 black, L8C9 black, L8C10 black.

9. C2: 74 compatible placements. L5C2 is filled.

10. C6: 6 compatible placements; L1C6 empty, L3C6 empty, L4C6 empty, L6C6 black.

11. C7: 15 compatible placements; L3C7 black, L4C7 black, L7C7 empty, L9C7 empty.

12. C9: 35 compatible placements; L3C9 black, L4C9 black.

13. C10: 20 compatible placements; L6C10 black, L7C10 black, L9C10 black, L10C10 black, L11C10 black.

14. L1: 20 compatible placements; L1C1 black, L1C2 empty, L1C3 black, L1C4 black, L1C5 black.

15. L3: 24 compatible placements; L3C8 empty, L3C10 empty.

16. L4: 3 compatible placements; L4C8 empty, L4C10 empty, L4C13 black, L4C14 empty, L4C16 black.

17. L6: 12 compatible placements; L6C1 empty, L6C4 black.

18. L7: 4 compatible placements; L7C1 black, L7C2 black, L7C3 empty, L7C4 black, L7C5 black, L7C8 empty, L7C13 black.

19. L9: 78 compatible placements. L9C11 is empty.

20. L11: 39 compatible placements. L11C12 is empty.

21. C1: 8 compatible placements. L2C1 is empty.

22. C2: 25 compatible placements; L2C2 empty, L6C2 black.

23. C3: 22 compatible placements; L2C3 black, L3C3 black, L4C3 black, L5C3 empty.

24. C4: 11 compatible placements; black L2C4, empty L3C4, empty L4C4.

25. C5: 18 compatible placements; L2C5 black, L3C5 empty, L8C5 black.

26. C8: 24 compatible placements. L1C8 is filled.

27. C10: 6 compatible placements; L12C10 black, L13C10 black.

28. L1: 16 compatible placements. L1C9 is filled.

29. L2: 4 compatible placements; L2C10 black, L2C11 black, L2C12 empty.

30. L3: 3 compatible placements; L3C1 empty, L3C2 black, L3C14 black, L3C15 black.

31. L4: 1 compatible placements; L4C1 empty, L4C5 black.

32. L5: 70 compatible placements. L5C1 is filled.

33. L6: 5 compatible placements; L6C3 black, L6C7 empty, L6C8 empty.

34. L8: 6 compatible placements; L8C3 empty, L8C6 empty, L8C11 black.

35. C1: 1 compatible placements; L8C1 black, L9C1 black, L10C1 black, L11C1 empty, L12C1 empty, L13C1 empty, L14C1 empty, L15C1 empty, L16C1 empty.

36. C2: 15 compatible placements. L8C2 is empty.

37. C5: 7 compatible placements; L5C5 black, L9C5 empty.

38. C6: 5 compatible placements. L5C6 is filled.

39. C9: 23 compatible placements. L5C9 is empty.

40. C10: 3 compatible placements. L1C10 is empty.

41. C11: 15 compatible placements; L1C11 black, L3C11 black, L5C11 empty, L7C11 empty.

42. C12: 74 compatible placements. L1C12 is empty.

43. C14: 56 compatible investments; L1C14 empty, L2C14 empty.

44. L1: 3 compatible placements. L1C7 is filled.

45. L3: 2 compatible placements. L3C12 is empty.

46. L5: 10 compatible placements; L5C4 empty, L5C7 empty.

47. L7: 3 compatible placements. L7C9 is filled.

48. L9: 18 compatible placements. L9C2 is empty.

49. L10: 39 compatible placements. L10C2 is empty.

50. L12: 86 compatible placements. L12C9 is empty.

51. C4: 5 compatible placements; L9C4 black, L10C4 empty.

52. C12: 29 compatible placements; L5C12 black, L6C12 empty.

53. L6: 3 compatible placements. L6C9 is filled.

54. L9: 17 compatible placements; L9C3 black, L9C9 empty.

55. C3: 1 compatible placements; L10C3 black, L11C3 black, L12C3 empty, L13C3 empty, L14C3 empty, L15C3 empty, L16C3 empty.

56. Case analysis on L3 : placement 1/2 = * * * * * * * * * * *

57. C13: 7 compatible placements; L1C13 empty, L2C13 empty, L5C13 empty, L12C13 black, L13C13 black.

58. L5: 4 compatible placements. L5C15 is filled.

59. L12: 25 compatible placements. L12C11 is empty.

60. C15: 50 compatible placements. L1C15 is empty.

61. L1: 1 compatible placements. L1C16 is filled.

62. C16: 15 compatible placements; L2C16 empty, L5C16 black, L6C16 empty.

63. L2: 1 compatible placements. L2C15 is filled.

64. L5: 2 compatible placements. L5C14 is empty.

65. L6: 2 compatible placements. L6C14 is filled.

66. C14: 15 compatible placements; L7C14 black, L8C14 empty.

67. C15: 35 compatible placements. L6C15 is empty.

68. L6: 1 compatible placements. L6C13 is filled.

69. L7: 1 compatible placements; L7C12 empty, L7C15 empty, L7C16 black.

70. L8: 1 compatible placements; L8C13 empty, L8C15 black, L8C16 black.

71. C15: 10 compatible placements. L9C15 is empty.

72. C16: 10 compatible placements. L9C16 is empty.

73. Case analysis on L5: placement 1/2 = ■·····

74. C8: 10 compatible placements; L9C8 black, L10C8 empty.

75. C10: 1 compatible placements; L14C10 black, L15C10 empty, L16C10 black.

76. L10: 15 compatible placements. L10C14 is filled.

77. L14: 33 compatible placements. L14C9 is empty.

78. C14: 4 compatible placements; L9C14 empty, L11C14 empty.

79. Case analysis on L9: placement 2/3 = '·····.

80. C6: 1 compatible placement; L10C6 black, L11C6 empty, L12C6 black, L13C6 empty, L14C6 black, L15C6 empty, L16C6 black.

81. C12: 6 compatible placements. L10C12 is empty.

82. C13: 3 compatible placements. L14C13 is filled.

83. L10: 4 compatible placements; L10C5 empty, L10C7 empty, L10C15 black.

84. L11: 8 compatible placements. L11C5 is empty.

85. L12: 9 compatible placements; L12C5 empty, L12C7 empty.

86. L14: 6 compatible placements; L14C5 empty, L14C7 empty, L14C11 empty, L14C14 black, L14C16 empty.

87. C14: 2 compatible placements; L12C14 empty, L16C14 empty.

88. C15: 6 compatible placements. L11C15 is empty.

89. Case analysis on C14: placement 2/2 =·····

90. Case analysis on C5 : placement 1/3 = ■^^^^^^^·····

91. Case analysis on C13: placement 2/3 = * * * * * * * * * * *

92. L10: 2 compatible placements. L10C16 is filled.

93. L11: 2 compatible placements; L11C7 empty, L11C8 empty, L11C11 black, L11C16 empty.

94. L15: 11 compatible placements. L15C16 is empty.

95. C11: 3 compatible placements. L10C11 is empty.

96. C16: 2 compatible placements. L16C16 is filled.

97. L10: 1 compatible placements. L10C9 is filled.

98. L16: 3 compatible placements; L16C2 empty, L16C4 empty, L16C7 empty, L16C15 empty.

99. C2: 3 compatible placements. L14C2 is filled.

100. Case analysis on L11: placement 1/2 = '·····

101. C2: 2 compatible placements. L12C2 is empty.

102. Case analysis on C2 : placement 1/2 = * * * * * * * * * * * *

103. L13: 5 compatible placements. L13C4 is filled.

104. C4: 2 compatible placements. L15C4 is empty.

105. Case analysis on C4: placement 2/2 = ■·····
·····
·····

106. L12: 1 compatible placements; L12C8 black, L12C12 empty, L12C15 black, L12C16 black.

107. L14: 2 compatible placements. L14C8 is empty.

108. C8: 2 compatible placements. L13C8 is empty.

109. C15: 2 compatible placements. L13C15 is empty.

110. C16: 1 compatible placements. L13C16 is empty.

111. L13: 2 compatible placements; L13C7 black, L13C12 empty.

112. C7: 1 compatible placements. L15C7 is empty.

113. C12: 1 compatible placements; L14C12 black, L15C12 empty, L16C12 black.

114. L14: 1 compatible placements. L14C15 is empty.

115. L15: 3 compatible placements. L15C15 is filled.

116. L16: 1 compatible placements; L16C8 empty, L16C9 empty, L16C11 black.

117. C8: 1 compatible placements. L15C8 is filled.

118. C11: 1 compatible placements; L13C11 empty, L15C11 empty.

119. L13: 1 compatible placements. L13C9 is filled.

120. L15: 1 compatible placements. L15C9 is empty.

121. Examples of contradictions encountered:

122. C15 has no legal placement left at depth 4.

123. C16 has no legal placement left at depth 5.

124. C15 has no legal placement left at depth 5.

125. C12 has no legal placement left at depth 5.

126. C15 has no legal placement left at depth 4.

127. C16 has no legal placement left at depth 6.

128. L13 has no legal placement left at depth 10.

129. C12 has no legal placement left at depth 10.

130. C4 has no legal placement left at depth 8.

131. C8 has no legal placement left at depth 7.

132. C16 has no legal placement left at depth 6.

133. C4 has no legal placement left at depth 5.

134. L14 has no legal placement left at depth 5.

135. C14 has no legal placement left at depth 3.

136. L10 has no legal placement left at depth 2.

137. C15 has no legal placement left at depth 1.

Common pitfalls

- Do not confuse an unknown box with an empty proven box.

- A hypothesis must be cancelled with all its consequences if a series becomes impossible.

- The form suggested by the first black boxes is not evidence.

Verification

Enumeration of placements by line, propagation by intersections and exhaustive search for up to two solutions; new verification of response series.

The Silent Checkpoints

Hamiltonian path / EXPERT

Answer

The full numbering below determines the only compatible route.

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	12	11	10	9	8	7
3	13	20	21	22	23	24
4	14	19	28	27	26	25
5	15	18	29	32	33	36
6	16	17	30	31	34	35

Reasoning

1. From marker 1 to marker 8, the portion retained is: 1 (L1C1) → 2 (L1C2) → 3 (L1C3) → 4 (L1C4) → 5 (L1C5) → 6 (L1C6) → 7 (L2C6) → 8 (L2C5). Check each orthogonal shift, then mark the boxes already consumed before continuing.

2. From marker 8 to marker 15, the portion retained is: 8 (L2C5) → 9 (L2C4) → 10 (L2C3) → 11 (L2C2) → 12 (L2C1) → 13 (L3C1) → 14 (L4C1) → 15 (L5C1). Check each orthogonal displacement, then mark the boxes already consumed before continuing.

3. From marker 15 to marker 18, the portion retained is: 15 (L5C1) → 16 (L6C1) → 17 (L6C2) → 18 (L5C2). Check each orthogonal shift, then mark the boxes already consumed before continuing.

4. From marker 18 to marker 28, the portion retained is: 18 (L5C2) → 19 (L4C2) → 20 (L3C2) → 21 (L3C3) → 22 (L3C4) → 23 (L3C5) → 24 (L3C6) → 25 (L4C6) → 26 (L4C5) → 27 (L4C4) → 28 (L4C3). Check each orthogonal shift, then mark the boxes already consumed before continuing.

5. From marker 28 to marker 33, the portion retained is: 28 (L4C3) → 29 (L5C3) → 30 (L6C3) → 31 (L6C4) → 32 (L5C4) → 33 (L5C5). Check each orthogonal shift, then mark the boxes already consumed before continuing.

6. From marker 33 to marker 35, the portion retained is: 33 (L5C5) → 34 (L6C5) → 35 (L6C6). Check each orthogonal shift, then mark the boxes already consumed before continuing.

7. From marker 35 to marker 36, the portion retained is: 35 (L6C6) → 36 (L5C6). Check each orthogonal shift, then mark the boxes already consumed before continuing.

8. The exploration of all compatible continuations gives 613 states. It rejects 46 separations of the remaining cells and 221 violations of distance or parity. Only one complete visit remains.

9. To reproduce this proof on paper, starting from 1, try each free neighbour in the order high, low, left, right, and keep a tree. Immediately reject a path that reaches a marker in the wrong rank, isolate an unvisited area or cannot reach the next marker in as many steps as required. The last complete path is that displayed; the validator provided performs exactly this exploration.

Common pitfalls

- Local continuity is not enough: a detour can cut the remaining boxes in half.

- Two boxes of the same color on a checker require an even number of movements; check this parity between markers.

- Reaching a landmark too early is as forbidden as missing it.

Verification

DFS comprehensive simple paths, with necessary controls of connection, benchmarks, distance and parity; comparison of all solutions up to two.

The One-Way Crossing

Hamiltonian path / EXPERT

Answer

The full numbering below determines the only compatible route.

	1	2	3	4	5	6
1	29	30	31	32	33	36
2	28	27	26	25	34	35
3	21	22	23	24	1	2
4	20	19	18	17	16	3
5	11	12	13	14	15	4
6	10	9	8	7	6	5

Reasoning

1. From marker 1 to marker 4, the portion retained is: 1 (L3C5) → 2 (L3C6) → 3 (L4C6) → 4 (L5C6). Check each orthogonal shift, then mark the boxes already consumed before continuing.

2. From marker 4 to marker 6, the portion retained is: 4 (L5C6) → 5 (L6C6) → 6 (L6C5). Check each orthogonal shift, then mark the boxes already consumed before continuing.

3. From marker 6 to marker 9, the portion retained is: 6 (L6C5) → 7 (L6C4) → 8 (L6C3) → 9 (L6C2). Check each orthogonal shift, then mark the boxes already consumed before continuing.

4. From marker 9 to marker 12, the portion retained is: 9 (L6C2) → 10 (L6C1) → 11 (L5C1) → 12 (L5C2). Check each orthogonal displacement, then mark the boxes already consumed before continuing.

5. From marker 12 to marker 21, the portion retained is: 12 (L5C2) → 13 (L5C3) → 14 (L5C4) → 15 (L5C5) → 16 (L4C5) → 17 (L4C4) → 18 (L4C3) → 19 (L4C2) → 20 (L4C1) → 21 (L3C1). Check each orthogonal shift, then mark the boxes already consumed before continuing.

6. From marker 21 to marker 28, the portion retained is: 21 (L3C1) → 22 (L3C2) → 23 (L3C3) → 24 (L3C4) → 25 (L2C4) → 26 (L2C3) → 27 (L2C2) → 28 (L2C1). Check each orthogonal shift, then mark the boxes already consumed before continuing.

7. From marker 28 to marker 36, the portion retained is: 28 (L2C1) → 29 (L1C1) → 30 (L1C2) → 31 (L1C3) → 32 (L1C4) → 33 (L1C5) → 34 (L2C5) → 35 (L2C6) → 36 (L1C6). Check each orthogonal shift, then mark the boxes already consumed before continuing.

8. The exploration of all compatible continuations gives 423 states. It rejects 93 separations of the remaining cells and 83 violations of distance or parity. Only one complete visit remains.

9. To reproduce this proof on paper, starting from 1, try each free neighbour in the order high, low, left, right, and keep a tree. Immediately reject a path that reaches a marker in the wrong rank, isolate an unvisited area or cannot reach the next marker in as many steps as required. The last complete path is that displayed; the validator provided performs exactly this exploration.

Common pitfalls

- Local continuity is not enough: a detour can cut the remaining boxes in half.

- Two boxes of the same color on a checker require an even number of movements; check this parity between markers.

- Reaching a landmark too early is as forbidden as missing it.

Verification

DFS comprehensive simple paths, with necessary controls of connection, benchmarks, distance and parity; comparison of all solutions up to two.

The Maze Without Walls

Hamiltonian path / EXTREME

Answer

The full numbering below determines the only compatible route.

	1	2	3	4	5	6
1	5	6	7	8	9	10
2	4	23	22	21	20	11
3	3	24	25	26	19	12
4	2	1	28	27	18	13
5	35	36	29	30	17	14
6	34	33	32	31	16	15

Reasoning

1. From marker 1 to marker 17, the portion retained is: 1 (L4C2) → 2 (L4C1) → 3 (L3C1) → 4 (L2C1) → 5 (L1C1) → 6 (L1C2) → 7 (L1C3) → 8 (L1C4) → 9 (L1C5) → 10 (L1C6) → 11 (L2C6) → 12 (L3C6) → 13 (L4C6) → 14 (L5C6) → 15 (L6C6) → 16 (L6C5) → 17 (L5C5). Check each orthogonal shift, then mark the boxes already consumed before continuing.

2. From marker 17 to marker 23, the portion retained is: 17 (L5C5) → 18 (L4C5) → 19 (L3C5) → 20 (L2C5) → 21 (L2C4) → 22 (L2C3) → 23 (L2C2). Check each orthogonal shift, then mark the boxes already consumed before continuing.

3. From marker 23 to marker 32, the portion retained is: 23 (L2C2) → 24 (L3C2) → 25 (L3C3) → 26 (L3C4) → 27 (L4C4) → 28 (L4C3) → 29 (L5C3) → 30 (L5C4) → 31 (L6C4) → 32 (L6C3). Check each orthogonal shift, then mark the boxes already consumed before continuing.

4. From marker 32 to marker 33, the portion retained is: 32 (L6C3) → 33 (L6C2). Check each orthogonal shift, then mark the boxes already consumed before continuing.

5. From marker 33 to marker 36, the portion retained is: 33 (L6C2) → 34 (L6C1) → 35 (L5C1) → 36 (L5C2). Check each orthogonal shift, then mark the boxes already consumed before continuing.

6. The exploration of all compatible continuations gives 5682 states. It rejects 1042 separations of the remaining cells and 856 violations of distance or parity. Only one complete visit remains.

7. To reproduce this proof on paper, starting from 1, try each free neighbour in the order high, low, left, right, and keep a tree. Immediately reject a path that reaches a marker in the wrong rank, isolate an unvisited area or cannot reach the next marker in as many steps as required. The last complete path is that displayed; the validator provided performs exactly this exploration.

Common pitfalls

- Local continuity is not enough: a detour can cut the remaining boxes in half.

- Two boxes of the same color on a checker require an even number of movements; check this parity between markers.

- Reaching a landmark too early is as forbidden as missing it.

Verification

DFS comprehensive simple paths, with necessary controls of connection, benchmarks, distance and parity; comparison of all solutions up to two.

The Eight Shapes

Pentomino tiling / EXPERT

Answer

The paving below is unique for the indicated pieces and clues. A letter identifies an entire pentomino.

	1	2	3	4	5	6	7	8
1	I	I	I	I	I	X	L	L
2	V	V	V	W	X	X	X	L
3	V	Z	Z	W	W	X	T	L
4	V	Z	P	P	W	W	T	L
5	Z	Z	P	P	P	T	T	T

Reasoning

1. The silhouettes are defined by the squares bearing their letter. The dots are not part of the room. Build possible orientations: four rotations, then their mirror, by removing the duplicates.

2. Start with the boxes bearing an index. For each orientation of the piece concerned, check that it covers all the indices of its letter and no indication of another letter. Then choose an empty box with the least possible investments.

3. Place I on L1C1, L1C2, L1C3, L1C4, L1C5. Verify that these five cells form a rotation or mirror of its silhouette and do not cover any parts already laid.

4. Place L on L1C7, L1C8, L2C8, L3C8, L4C8. Verify that these five cells form a rotation or mirror of its silhouette and do not cover any rooms already laid.

5. Place T on L3C7, L4C7, L5C6, L5C7, L5C8. Verify that these five boxes form a rotation or mirror of its silhouette and do not cover any rooms already laid.

6. Place X on L1C6, L2C5, L2C6, L2C7, L3C6. Verify that these five boxes form a rotation or mirror of its silhouette and do not cover any parts already laid.

7. Place W on L2C4, L3C4, L3C5, L4C5, L4C6. Verify that these five cells form a rotation or mirror of its silhouette and do not cover any parts already laid.

8. Place P on L4C3, L4C4, L5C3, L5C4, L5C5. Verify that these five cells form a rotation or mirror of its silhouette and do not cover any rooms already laid.

9. Place Z on L3C2, L3C3, L4C2, L5C1, L5C2. Verify that these five cells form a rotation or mirror of its silhouette and do not cover any parts already laid.

10. Place V on L2C1, L2C2, L2C3, L3C1, L4C1. Verify that these five cells form a rotation or mirror of its silhouette and do not cover any parts already laid.

11. The proof of uniqueness explores 980 subpaves and eliminates 634 dead ends where a free box no longer accepts any remaining parts. Only one complete cover remains.

12. To independently check the answer, count five boxes per letter, compare each group to its close rotation/turn silhouette, and then check the clues. To prove uniqueness on paper, take the placement tree by the most constrained box and bar each branch that leaves an isolated box or impossible to cover. The validation script performs this exhaustive exploration; the displayed groups give the winning path.

Common pitfalls

- A mirror is allowed, but not a partial displacement of only one box.

- Two groups of the same range are not necessarily of the same form.

- A cavity of five cells may remain unfilled if its shape does not correspond to any remaining room.

Verification

Accurate coverage of boxes and coin identities, all free orientations; counting up to two solutions and independent verification of groups of five boxes.

The Locked Mosaic

Pentomino tiling / EXTREME

Answer

The paving below is unique for the indicated pieces and clues. A letter identifies an entire pentomino.

	1	2	3	4	5	6	7	8	9	10
1	L	L	X	U	U	W	Y	Y	Y	Y
2	L	X	X	X	U	W	W	Y	Z	Z
3	L	T	X	U	U	F	W	W	Z	V
4	L	T	N	N	F	F	F	Z	Z	V
5	T	T	T	N	N	N	F	V	V	V

Reasoning

1. The silhouettes are defined by the squares bearing their letter. The dots are not part of the room. Build possible orientations: four rotations, then their mirror, by removing the duplicates.

2. Start with the boxes bearing an index. For each orientation of the piece concerned, check that it covers all the indices of its letter and no indication of another letter. Then choose an empty box with the least possible investments.

3. Place F on L3C6, L4C5, L4C6, L4C7, L5C7. Verify that these five boxes form a rotation or mirror of its silhouette and do not cover any room already laid.

4. Place N on L4C3, L4C4, L5C4, L5C5, L5C6. Verify that these five cells form a rotation or mirror of its silhouette and do not cover any rooms already laid.

5. Place T on L3C2, L4C2, L5C1, L5C2, L5C3. Verify that these five boxes form a rotation or mirror of its silhouette and do not cover any rooms already laid.

6. Place L on L1C1, L1C2, L2C1, L3C1, L4C1. Verify that these five cells form a rotation or mirror of its silhouette and do not cover any parts already laid.

7. Place X on L1C3, L2C2, L2C3, L2C4, L3C3. Verify that these five boxes form a rotation or mirror of its silhouette and do not cover any room already laid.

8. Place U on L1C4, L1C5, L2C5, L3C4, L3C5. Verify that these five cells form a rotation or mirror of its silhouette and do not cover any parts already laid.

9. Place W on L1C6, L2C6, L2C7, L3C7, L3C8. Verify that these five cells form a rotation or mirror of its silhouette and do not cover any rooms already laid.

10. Place Y on L1C7, L1C8, L1C9, L1C10, L2C8. Verify that these five cells form a rotation or mirror of its silhouette and do not cover any rooms already laid.

11. Place V on L3C10, L4C10, L5C8, L5C9, L5C10. Verify that these five cells form a rotation or mirror of its silhouette and do not cover any rooms already laid.

12. Place Z on L2C9, L2C10, L3C9, L4C8, L4C9. Verify that these five boxes form a rotation or mirror of its silhouette and do not cover any room already laid.

13. The proof of uniqueness explores 6105 subpaves and eliminates 4131 dead ends where a free box no longer accepts any remaining parts. Only one complete cover remains.

14. To independently check the answer, count five boxes per letter, compare each group to its close rotation/turn silhouette, and then check the clues. To prove uniqueness on paper, take the placement tree by the most constrained box and bar each branch that leaves an isolated box or impossible to cover. The validation script performs this exhaustive exploration; the displayed groups give the winning path.

Common pitfalls

- A mirror is allowed, but not a partial displacement of only one box.

- Two groups of the same range are not necessarily of the same form.

- A cavity of five cells may remain unfilled if its shape does not correspond to any remaining room.

Verification

Accurate coverage of boxes and coin identities, all free orientations; counting up to two solutions and independent verification of groups of five boxes.

The Stubborn Rectangle

Pentomino tiling / EXTREME

Answer

The paving below is unique for the indicated pieces and clues. A letter identifies an entire pentomino.

	1	2	3	4	5	6	7	8	9	10
1	F	F	Z	Z	P	P	V	V	V	I
2	L	F	F	Z	P	P	P	N	V	I
3	L	F	X	Z	Z	W	T	N	V	I
4	L	X	X	X	W	W	T	N	N	I
5	L	L	X	W	W	T	T	T	N	I

Reasoning

1. The silhouettes are defined by the squares bearing their letter. The dots are not part of the room. Build possible orientations: four rotations, then their mirror, by removing the duplicates.

2. Start with the boxes bearing an index. For each orientation of the piece concerned, check that it covers all the indices of its letter and no indication of another letter. Then choose an empty box with the least possible investments.

3. Place T on L3C7, L4C7, L5C6, L5C7, L5C8. Verify that these five boxes form a rotation or mirror of its silhouette and do not cover any rooms already laid.

4. Place I on L1C10, L2C10, L3C10, L4C10, L5C10. Verify that these five cells form a rotation or mirror of its silhouette and do not cover any parts already laid.

5. Place N on L2C8, L3C8, L4C8, L4C9, L5C9. Verify that these five cells form a rotation or mirror of its silhouette and do not cover any rooms already laid.

6. Place V on L1C7, L1C8, L1C9, L2C9, L3C9. Verify that these five cells form a rotation or mirror of its silhouette and do not cover any rooms already laid.

7. Place P on L1C5, L1C6, L2C5, L2C6, L2C7. Verify that these five cells form a rotation or mirror of its silhouette and do not cover any rooms already laid.

8. Place F on L1C1, L1C2, L2C2, L2C3, L3C2. Verify that these five cells form a rotation or mirror of its silhouette and do not cover any rooms already laid.

9. Place L on L2C1, L3C1, L4C1, L5C1, L5C2. Verify that these five cells form a rotation or mirror of its silhouette and do not cover any parts already laid.

10. Place Z on L1C3, L1C4, L2C4, L3C4, L3C5. Verify that these five cells form a rotation or mirror of its silhouette and do not cover any rooms already laid.

11. Place W on L3C6, L4C5, L4C6, L5C4, L5C5. Verify that these five cells form a rotation or mirror of its silhouette and do not cover any rooms already laid.

12. Place X on L3C3, L4C2, L4C3, L4C4, L5C3. Verify that these five cells form a rotation or mirror of its silhouette and do not cover any parts already laid.

13. The proof of uniqueness explores 13455 subpaves and eliminates 8821 dead ends where a free box no longer accepts any remaining parts. Only one complete cover remains.

14. To independently check the answer, count five boxes per letter, compare each group to its close rotation/turn silhouette, and then check the clues. To prove uniqueness on paper, take the placement tree by the most constrained box and bar each branch that leaves an isolated box or impossible to cover. The validation script performs this exhaustive exploration; the displayed groups give the winning path.

Common pitfalls

- A mirror is allowed, but not a partial displacement of only one box.

- Two groups of the same range are not necessarily of the same form.

- A cavity of five cells may remain unfilled if its shape does not correspond to any remaining room.

Verification

Accurate coverage of boxes and coin identities, all free orientations; counting up to two solutions and independent verification of groups of five boxes.

The Final Tempo

Othello / Reversi / EXPERT

Answer

Black must play E8: differential guaranteed +6.

Reasoning

1. Start by establishing the legal blows by looking, in each of the eight directions, for an unempty adverse suite ended with a pawn of your color. A blow in a corner is not automatically the best.

2. Each sheet is evaluated by the number of pawns of the camp at the beginning minus those of the other camp. At a node where this camp plays, one keeps the maximum; at an opposing node, the minimum. A pass changes the line without consuming any box; two camps without legal blow complete the part.

3. The exact values of the first shots, after the best defence: E1: +2; E2: -4; A6: +2; A7: -4; B7: -8; C8: -10; E8: +6. The maximum is unique: E8.

4. After E8, all the opposing legal answers must be dealt with. Here is an optimal continuation of the two camps (N = Black, B = White); the score is always brought back to the camp that was to play at the start:

5. Adverse response E1: N E2, B C8, N A7, B B7, N A8, B A6. End: 37 black, 27 white; initial differential +10.

6. Adverse response E2: N E1, B B7, N A8, B C8, N A7, B A6. End: 40 black, 24 white; initial differential +16.

7. Adverse response A6: N C8, B B7, N A7, B A8, N E2, B E1. End: 35 black, 29 white; initial differential +6.

8. Adverse response B7: N A6, B C8, N E2, B A7, N A8, B E1. End: 35 black, 29 white; initial differential +6.

9. Adverse response C8: N E1, B E2, N A7, B B7, N A8, B A6. End: 37 black, 27 white; initial differential +10.

10. The opponent retains the answer giving the smallest of these differentials: 6. The other answers therefore do not refute the first blow; they obtain an equal or better result for you.

11. Complete main line: N E8, B A6, N C8, B B7, N A7, B A8, N E2, B E1. Final check: 35 blacks and 29 whites.

12. Technique: Minimal retrograde calculation, mobility and pass management, then comparison of filling orders. Note for each branch available blows rather than assuming an uninterrupted alternation. The table of values is an exhaustive search-assisted proof; continuations alone would not prove that the omitted defences are inferior.

Common pitfalls

- Count only the pawns returned by the next shot.

- Forget a diagonal or a mandatory pass.

- Consolidate a main line with evidence against all defences.

Verification

Legal part generation from the standard initial position; exhaustive minima stored to the end, without truncated depth or leaf heuristic. All the values of the first strokes are checked.

The Deceptive Corner

Othello / Reversi / EXPERT

Answer

Black must play G1: differential guaranteed +24.

Reasoning

1. Start by establishing the legal blows by looking, in each of the eight directions, for an unempty adverse suite ended with a pawn of your color. A blow in a corner is not automatically the best.

2. Each sheet is evaluated by the number of pawns of the camp at the beginning minus those of the other camp. At a node where this camp plays, one keeps the maximum; at an opposing node, the minimum. A pass changes the line without consuming any box; two camps without legal blow complete the part.

3. The exact values of the first shots, after the best defense: G1: +24; A3: +22; H3: +14; E7: +6; F7: +0; E8: +6; G8: -4. The maximum is unique: G1.

4. After G1, all opposing legal responses must be addressed. Here is an optimal continuation of the two camps (N = Black, B = White); the score is always brought back to the camp that was to play at the start:

5. Adverse response H3: N A3, B E8, N F8, B passes, N E7, B F7, N G8. End: 44 black, 20 white; initial differential +24.

6. Adverse response E8: N H3, B passes, N A3, B passes, N F8, B passes, N E7, B F7, N G8. Finish: 46 black, 18 white; initial differential +28.

7. The opponent holds the answer giving the smallest of these differentials: 24. The other answers therefore do not refute the first blow; they obtain an equal or better result for you.

8. Full main line: N G1, B H3, N A3, B E8, N F8, B pass, N E7, B F7, N G8. Final check: 44 black and 20 white. The blanks are not assigned to anyone.

9. Technique: Minimal retrograde calculation, mobility and pass management, then comparison of filling orders. Note for each branch available blows rather than assuming an uninterrupted alternation. The table of values is an exhaustive search-assisted proof; continuations alone would not prove that the omitted defences are inferior.

Common pitfalls

- Count only the pawns returned by the next shot.

- Forget a diagonal or a mandatory pass.

- Consolidate a main line with evidence against all defences.

Verification

Legal part generation from the standard initial position; exhaustive minima stored to the end, without truncated depth or leaf heuristic. All the values of the first strokes are checked.

Mobility in Reserve

Othello / Reversi / EXPERT

Answer

White must play H1: differential guaranteed +16.

Reasoning

1. Start by establishing the legal blows by looking, in each of the eight directions, for an unempty adverse suite ended with a pawn of your color. A blow in a corner is not automatically the best.

2. Each sheet is evaluated by the number of pawns of the camp at the beginning minus those of the other camp. At a node where this camp plays, one keeps the maximum; at an opposing node, the minimum. A pass changes the line without consuming any box; two camps without legal blow complete the part.

3. The exact values of the first shots, after the best defence: H1: +16; A2: -20; A3: -16; G7: +0; A8: -12; C8: -22; G8: +2. The maximum is unique: H1.

4. After H1, all the opposing legal answers must be processed. Here is an optimal continuation of the two camps (N = Black, B = White); the score is always brought back to the camp which was to play at the start:

5. Adverse response H4: B A8, N G7, B G8, N D8, B A3, N A2, B C8. End: 24 black, 40 white; initial differential +16.

6. Opposite response G7: B H4, N D8, B A8, N pass, B C8, N pass, B G8, N pass, B A3, N A2. Finish: 22 black, 42 white; initial differential +20.

7. Adverse response C8: B H4, N D8, B A8, N G7, B G8, N pass, B A3, N pass, B A2. Finish: 15 black, 49 white; initial differential +34.

8. Adverse response D8: B H4, N G7, B A8, N pass, B C8, N pass, B G8, N pass, B A3, N A2. End: 22 black, 42 white; initial differential +20.

9. The opponent retains the answer giving the smallest of these differentials: 16. The other answers therefore do not refute the first blow; they obtain an equal or better result for you.

10. Complete main line: B H1, N H4, B A8, N G7, B G8, N D8, B A3, N A2, B C8. Final check: 24 black and 40 white. The blank boxes are not assigned to anyone.

11. Technique: Minimal retrograde calculation, mobility and pass management, then comparison of filling orders. Note for each branch available blows rather than assuming an uninterrupted alternation. The table of values is an exhaustive search-assisted proof; continuations alone would not prove that the omitted defences are inferior.

Common pitfalls

- Count only the pawns returned by the next shot.

- Forget a diagonal or a mandatory pass.

- Consolidate a main line with evidence against all defences.

Verification

Legal part generation from the standard initial position; exhaustive minima stored to the end, without truncated depth or leaf heuristic. All the values of the first strokes are checked.

The Defense Tree

Othello / Reversi / EXTREME

Answer

Black must play H7: differential guaranteed +12.

Reasoning

1. Start by establishing the legal blows by looking, in each of the eight directions, for an unempty adverse suite ended with a pawn of your color. A blow in a corner is not automatically the best.

2. Each sheet is evaluated by the number of pawns of the camp at the beginning minus those of the other camp. At a node where this camp plays, one keeps the maximum; at an opposing node, the minimum. A pass changes the line without consuming any box; two camps without legal blow complete the part.

3. The exact values of the first shots, after the best defense: D1: -10; E2: -14; A3: -14; A6: +4; A7: +4; H7: +12. The maximum is unique: H7.

4. After H7, all the opposing legal answers must be dealt with. Here is an optimal continuation of the two camps (N = Black, B = White); the score is always brought back to the camp which was to play at the start:

5. Adverse response D1: N A3, B E2, N A7, B A6, N G2, B G1, N F1, B passes, N F2. Finish: 45 black, 19 white; initial differential +26.

6. Adverse response E2: N D1, B A3, N F2, B F1, N A6, B A7, N G1, B G2. Finish: 38 black, 26 white; initial differential +12.

7. Opposite response G2: N A3, B D1, N E2, B F2, N G1, B A6, N F1, B passes, N A7. Finish: 45 black, 19 white; initial differential +26.

8. Adverse response A3: N D1, B E2, N A6, B G2, N G1, B A7, N F2, B passes, N F1. Finish: 40 black, 24 white; initial differential +16.

9. Adverse response A6: N A3, B F2, N D1, B E2, N A7, B G2, N G1, B F1. Finish: 44 black, 20 white; initial differential +24.

10. The adversary retains the answer giving the smallest of these differentials: 12. The other answers therefore do not refute the first blow; they obtain an equal or better result for you.

11. Complete main line: N H7, B E2, N D1, B A3, N F2, B F1, N A6, B A7, N G1, B G2. Final check: 38 blacks and 26 whites.

12. Technique: Minimal retrograde calculation, mobility and pass management, then comparison of filling orders. Note for each branch available blows rather than assuming an uninterrupted alternation. The table of values is an exhaustive search-assisted proof; continuations alone would not prove that the omitted defences are inferior.

Common pitfalls

- Count only the pawns returned by the next shot.

- Forget a diagonal or a mandatory pass.

- Consolidate a main line with evidence against all defences.

Verification

Legal part generation from the standard initial position; exhaustive minima stored to the end, without truncated depth or leaf heuristic. All the values of the first strokes are checked.

Paths to a Majority

Othello / Reversi / EXTREME

Answer

White must play H7: differential guaranteed +16.

Reasoning

1. Start by establishing the legal blows by looking, in each of the eight directions, for an unempty adverse suite ended with a pawn of your color. A blow in a corner is not automatically the best.

2. Each sheet is evaluated by the number of pawns of the camp at the beginning minus those of the other camp. At a node where this camp plays, one keeps the maximum; at an opposing node, the minimum. A pass changes the line without consuming any box; two camps without legal blow complete the part.

3. The exact values of the first shots, after the best defense: G3: +12; G5: +8; H7: +16; C8: +10; G8: +12. The maximum is unique: H7.

4. After H7, all the opposing legal answers must be dealt with. Here is an optimal continuation of the two camps (N = Black, B = White); the score is always brought back to the camp which was to play at the start:

5. Adverse response A1: B G5, N B1, B A2, N G3, B C8, N H8, B G8, N H3, B H2. Finish: 19 black, 45 white; initial differential +26.

6. Adverse response B1: B A2, N A1, B G5, N G3, B C8, N H8, B G8, N H3, B H2. End: 20 black, 44 white; initial differential +24.

7. Adverse response A2: B B1, N A1, B G5, N G3, B C8, N H8, B G8, N H3, B H2. Finish: 19 black, 45 white; initial differential +26.

8. Adverse response H2: B H3, N G5, B G3, N H8, B C8, N A1, B G8, N B1, B A2. End: 20 black, 44 white; initial differential +24.

9. Adverse response H3: B H2, N H8, B C8, N G5, B G8, N A1, B G3, N B1, B A2. Finish: 15 black, 49 white; initial differential +34.

10. Opposite response G5: B C8, N H2, B G3, N H3, B G8, N A1, B B1, N A2, B passes, N H8. End: 24 black, 40 white; initial differential +16.

11. Adverse response H8: B G5, N G3, B C8, N H3, B H2, N A1, B G8, N B1, B A2. End: 20 black, 44 white; initial differential +24.

12. The opponent retains the answer giving the smallest of these differentials: 16. The other answers therefore do not refute the first blow; they obtain an equal or better result for you.

13. Complete main line: B H7, N G5, B C8, N H2, B G3, N H3, B G8, N A1, B B1, N A2, B passes, N H8. Final check: 24 blacks and 40 whites.

14. Technique: Minimal retrograde calculation, mobility and pass management, then comparison of filling orders. Note for each branch available blows rather than assuming an uninterrupted alternation. The table of values is an exhaustive search-assisted proof; continuations alone would not prove that the omitted defences are inferior.

Common pitfalls

- Count only the pawns returned by the next shot.

- Forget a diagonal or a mandatory pass.

- Consolidate a main line with evidence against all defences.

Verification

Legal part generation from the standard initial position; exhaustive minima stored to the end, without truncated depth or leaf heuristic. All the values of the first strokes are checked.

The Last Word

Othello / Reversi / EXTREME

Answer

White must play A8: guaranteed differential +6.

Reasoning

1. Start by establishing the legal blows by looking, in each of the eight directions, for an unempty adverse suite ended with a pawn of your color. A blow in a corner is not automatically the best.

2. Each sheet is evaluated by the number of pawns of the camp at the beginning minus those of the other camp. At a node where this camp plays, one keeps the maximum; at an opposing node, the minimum. A pass changes the line without consuming any box; two camps without legal blow complete the part.

3. The exact values of the first shots, after the best defense: A1: -6; C1: +4; A2: -30; A8: +6; B8: -2; F8: -6. The maximum is unique: A8.

4. After A8, you have to deal with all the opposing legal answers. Here is an optimal continuation of the two camps (N = Black, B = White); the score is always brought back to the camp that was to play at the start:

5. Adverse response C1: B B8, N F8, B A1, N H1, B A2, N G1, B H2, N H4, B H5. End: 29 black, 35 white; initial differential +6.

6. Opposite response G1: B B8, N C1, B A1, N A2, B H1, N H2, B H4, N F8, B H5. End: 22 black, 42 white; initial differential +20.

7. Adverse response H2: B B8, N F8, B A2, N H4, B H5, N pass, B A1, N pass, B C1, N pass, B G1, N pass, B H1. End: 16 black, 48 white; initial differential +32.

8. Adverse response H4: B B8, N F8, B H5, N G1, B A1, N A2, B C1, N pass, B H1, N H2. End: 24 black, 40 white; initial differential +16.

9. Adverse response H5: B B8, N H4, B A1, N F8, B A2, N G1, B C1, N pass, B H1, N H2. End: 27 black, 37 white; initial differential +10.

10. Adverse response B8: B F8, N H1, B C1, N A1, B A2, N G1, B H2, N H5, B H4. End: 28 black, 36 white; initial differential +8.

11. Adverse response F8: B B8, N H5, B H4, N C1, B A1, N G1, B H1, N H2, B A2. Finish: 26 black, 38 white; initial differential +12.

12. The opponent retains the answer giving the smallest of these differentials: 6. The other answers therefore do not refute the first blow; they obtain an equal or better result for you.

13. Complete main line: B A8, N C1, B B8, N F8, B A1, N H1, B A2, N G1, B H2, N H4, B H5. Final check: 29 blacks and 35 whites. The blanks are not assigned to anyone.

14. Technique: Minimal retrograde calculation, mobility and pass management, then comparison of filling orders. Note for each branch available blows rather than assuming an uninterrupted alternation. The table of values is an exhaustive search-assisted proof; continuations alone would not prove that the omitted defences are inferior.

Common pitfalls

- Count only the pawns returned by the next shot.

- Forget a diagonal or a mandatory pass.

- Consolidate a main line with evidence against all defences.

Verification

Legal part generation from the standard initial position; exhaustive minima stored to the end, without truncated depth or leaf heuristic. All the values of the first strokes are checked.

Two Workshops

Nim — pair subtraction / EXPERT

Answer

Winning shots: 1: A−1 and B−1 → (7, 10); 1: A−3 → (5, 11); 2: B−3 → (10, 4).

Reasoning

1. Each pair is a finished impartial game: the total number of chips decreases strictly at each stroke. At the position (0, 0), no movement exists; its number of Grundy is 0.

2. For a pair (a, b), build all the numbers that can be reached by removing 1, 2 or 3 A alone, B alone, or both simultaneously. The number of Grundy is the mex: the smallest non-negative integer absent from this set. Calculates by increasing amounts a+b, as all successors have a smaller sum.

3. Start of the table, useful for checking the convention: a\b | 0 1 2 3 4 5 6 7 ; 0 | 0 1 2 3 0 1 2 3 ; 1 | 1 2 0 4 1 2 0 4 ; 2 | 2 0 1 5 3 0 1 2 ; 3 | 3 4 5 6 2 7 4 5 ; 4 | 0 1 3 2 0 1 5 3 ; 5 | 1 2 0 7 1 2 0 4 ; 6 | 2 0 1 4 5 0 1 6 ; 7 | 3 4 2 5 3 4 6 5.

4. The values of the given pairs are: g(8,11)=2, g(10,7)=4. Their XOR is 6. It is not null: the position is winning, provided you join a null XOR.

5. For each move in pair k, replace its old value g with the new value g′: new XOR = old XOR XOR g XOR g′. The winning moves are exactly: 1: A−1 and B−1 → (7, 10); 1: A−3 → (5, 11); 2: B−3 → (10, 4).

6. Control of all other hits: 1: A−1 → (7, 11) : XOR 1 ; 1: B−1 → (8, 10) : XOR 1 ; 1: A−2 → (6, 11) : XOR 3 ; 1: B−2 → (8, 9) : XOR 5 ; 1: A−2 and B−2 → (6, 9) : XOR 4 ; 1: B−3 → (8, 8) : XOR 4 ; 1: A−3 and B−3 → (5, 8) : XOR 5 ; 2: A−1 → (9, 7) : XOR 5 ; 2: B−1 → (10, 6) : XOR 3 ; 2: A−1 and B−1 → (9, 6) : XOR 2 ; 2: A−2 → (8, 7) : XOR 1 ; 2: B−2 → (10, 5) : XOR 2 ; 2: A−2 and B−2 → (8, 5) : XOR 3 ; 2: A−3 → (7, 7) : XOR 7 ; 2: A−3 and B−3 → (7, 4) : XOR 1. winner exists for the opponent.

7. Why this works: when the XOR of the components is zero, a move changes one Grundy number to a different value, making the XOR nonzero. When the XOR is nonzero, choose a component with the highest set bit of the XOR. Its target g′ = g XOR XOR is less than g. By the definition of mex, every value below g is reachable. This lets you restore the XOR to zero after each opposing move.

8. The game ends because the total number of tokens decreases. Keeping the XOR at zero after each of your moves ensures you are never the player unable to move: (0,0) for every pair gives XOR zero. You therefore take the final token. This reasoning covers every opponent response.

9. For the verification of high values, apply exactly the recurrence, up to a maximum of 11 of this exercise. A symmetry g(a,b)=g(b,a) is permitted. Does not assume periodicity from the first eight lines: the evidence here uses the complete table.

Common pitfalls

- Apply the XOR of the heap sizes: it is the Grundy numbers of the PAIRS that combine.

- Forget simultaneous withdrawals, or allow two different quantities when withdrawing simultaneously.

- Transpose a poverty rule to this normal rule.

Verification

Accurate dynamic programming of Grundy numbers, control of the mex and all the first moves. The sum property is demonstrated in the fix.

The Broken Balance

Nim — pair subtraction / EXPERT

Answer

Winning shots: 2: A−3 → (4, 13).

Reasoning

1. Each pair is a finished impartial game: the total number of chips decreases strictly at each stroke. At the position (0, 0), no movement exists; its number of Grundy is 0.

2. For a pair (a, b), build all the numbers that can be reached by removing 1, 2 or 3 A alone, B alone, or both simultaneously. The number of Grundy is the mex: the smallest non-negative integer absent from this set. Calculates by increasing amounts a+b, as all successors have a smaller sum.

3. Start of the table, useful for checking the convention: a\b | 0 1 2 3 4 5 6 7 ; 0 | 0 1 2 3 0 1 2 3 ; 1 | 1 2 0 4 1 2 0 4 ; 2 | 2 0 1 5 3 0 1 2 ; 3 | 3 4 5 6 2 7 4 5 ; 4 | 0 1 3 2 0 1 5 3 ; 5 | 1 2 0 7 1 2 0 4 ; 6 | 2 0 1 4 5 0 1 6 ; 7 | 3 4 2 5 3 4 6 5.

4. The values of the given pairs are: g(12,9)=1, g(7,13)=6. Their XOR is 7. It is not null: the position is winning, provided you join a null XOR.

5. For each move in pair k, replace its old value g with the new value g′: new XOR = old XOR XOR g XOR g′. The only winning move is 2: A−3 → (4, 13).

6. Check of all other moves: 1: A−1 → (11, 9) : XOR 5 ; 1: B−1 → (12, 8) : XOR 6 ; 1: A−1 and B−1 → (11, 8) : XOR 4 ; 1: A−2 → (10, 9) : XOR 6 ; 1: B−2 → (12, 7) : XOR 4 ; 1: A−2 and B−2 → (10, 7) : XOR 2 ; 1: A−3 → (9, 9) : XOR 4 ; 1: B−3 → (12, 6) : XOR 5 ; 1: A−3 and B−3 → (9, 6) : XOR 6 ; 2: A−1 → (6, 13) : XOR 1 ; 2: B−1 → (7, 12) : XOR 3 ; 2: A−1 and B−1 → (6, 12) : XOR 2 ; 2: A−2 → (5, 13) : XOR 3 ; 2: B−2 → (7, 11) : XOR 4 ; 2: A−2 and B−2 → (5, 11) : XOR 5 ; 2: B−3 → (7, 10) : XOR 5 ; 2: A−3 and B−3 → (4, 10) : XOR 3. They each leave a non-zero sum: a winning answer exists for the opponent.

7. Why this works: when the XOR of the components is zero, a move changes one Grundy number to a different value, making the XOR nonzero. When the XOR is nonzero, choose a component with the highest set bit of the XOR. Its target g′ = g XOR XOR is less than g. By the definition of mex, every value below g is reachable. This lets you restore the XOR to zero after each opposing move.

8. The game ends because the total number of tokens decreases. Keeping the XOR at zero after each of your moves ensures you are never the player unable to move: (0,0) for every pair gives XOR zero. You therefore take the final token. This reasoning covers every opponent response.

9. For the verification of high values, apply exactly the recurrence, up to a maximum of 13 of this exercise. A symmetry g(a,b)=g(b,a) is allowed. Does not assume periodicity from the first eight lines: the evidence here uses the complete table.

Common pitfalls

- Apply the XOR of the heap sizes: it is the Grundy numbers of the PAIRS that combine.

- Forget simultaneous withdrawals, or allow two different quantities when withdrawing simultaneously.

- Transpose a poverty rule to this normal rule.

Verification

Accurate dynamic programming of Grundy numbers, control of the mex and all the first moves. The sum property is demonstrated in the fix.

Three Linked Fronts

Nim — pair subtraction / EXTREME

Answer

Winning shots: 1: A−3 → (14, 19); 2: A−3 → (11, 18).

Reasoning

1. Each pair is a finished impartial game: the total number of chips decreases strictly at each stroke. At the position (0, 0), no movement exists; its number of Grundy is 0.

2. For a pair (a, b), build all the numbers that can be reached by removing 1, 2 or 3 A alone, B alone, or both simultaneously. The number of Grundy is the mex: the smallest non-negative integer absent from this set. Calculates by increasing amounts a+b, as all successors have a smaller sum.

3. Start of the table, useful for checking the convention: a\b | 0 1 2 3 4 5 6 7 ; 0 | 0 1 2 3 0 1 2 3 ; 1 | 1 2 0 4 1 2 0 4 ; 2 | 2 0 1 5 3 0 1 2 ; 3 | 3 4 5 6 2 7 4 5 ; 4 | 0 1 3 2 0 1 5 3 ; 5 | 1 2 0 7 1 2 0 4 ; 6 | 2 0 1 4 5 0 1 6 ; 7 | 3 4 2 5 3 4 6 5.

4. The values of the given pairs are: g(17,19)=5, g(14,18)=1, g(20,15)=2. Their XOR is 6. It is non-zero: the position is winning, provided you join a null XOR.

5. For each move in pair k, replace its old value g with the new value g′: new XOR = old XOR XOR g XOR g′. The winning moves are exactly: 1: A−3 → (14, 19); 2: A−3 → (11, 18).

6. Control of all other hits: 1: A−1 → (16, 19) : XOR 1 ; 1: B−1 → (17, 18) : XOR 3 ; 1: A−1 and B−1 → (16, 18) : XOR 7 ; 1: A−2 → (15, 19) : XOR 11 ; 1: B−2 → (17, 17) : XOR 1 ; 1: A−2 and B−2 → (15, 17) : XOR 4 ; 1: B−3 → (17, 16) : XOR 2 ; 1: A−3 and B−3 → (14, 16) : XOR 1 ; 2: A−1 → (13, 18) : XOR 7 ; 2: B−1 → (14, 17) : XOR 7 ; 2: A−1 and B−1 → (13, 17) : XOR 5 ; 2: A−2 → (12, 18) : XOR 4 ; 2: B−2 → (14, 16) : XOR 5 ; 2: A−2 and B−2 → (12, 16) : XOR 7 ; 2: B−3 → (14, 15) : XOR 1 ; 2: A−3 and B−3 → (11, 15) XOR 2 ; 3: A−1 → (19, 15) : XOR 12 ; 3: B−1 → (20, 14) : XOR 1 ; 3: A−1 and B−1 → (19, 14) : XOR 7 ; 3: A−2 → (18, 15) : XOR 2 ; 3: B−2 → (20, 13) : XOR 5 ; 3: A−2 and B−2 → (18, 13) : XOR 4 ; 3: A−3 → (17, 15) : XOR 3 ; 3: B−3 → (20, 12) : XOR 4 ; 3: A−3 and B−3 → (17, 12) : XOR 5. They each leave a non-zero sum: a winning answer exists for the adversary.

7. Why this works: when the XOR of the components is zero, a move changes one Grundy number to a different value, making the XOR nonzero. When the XOR is nonzero, choose a component with the highest set bit of the XOR. Its target g′ = g XOR XOR is less than g. By the definition of mex, every value below g is reachable. This lets you restore the XOR to zero after each opposing move.

8. The game ends because the total number of tokens decreases. Keeping the XOR at zero after each of your moves ensures you are never the player unable to move: (0,0) for every pair gives XOR zero. You therefore take the final token. This reasoning covers every opponent response.

9. For the verification of high values, apply exactly the recurrence, up to a maximum of 20 of this exercise. A symmetry g(a,b)=g(b,a) is allowed. Does not assume periodicity from the first eight lines: the evidence here uses the complete table.

Common pitfalls

- Apply the XOR of the heap sizes: it is the Grundy numbers of the PAIRS that combine.

- Forget simultaneous withdrawals, or allow two different quantities when withdrawing simultaneously.

- Transpose a poverty rule to this normal rule.

Verification

Accurate dynamic programming of Grundy numbers, control of the mex and all the first moves. The sum property is demonstrated in the fix.

The Great Pile Board

Nim — pair subtraction / EXTREME

Answer

Winning shots: 1: B−1 → (21, 15); 2: A−2 → (16, 23); 2: A−3 and B−3 → (15, 20).

Reasoning

1. Each pair is a finished impartial game: the total number of chips decreases strictly at each stroke. At the position (0, 0), no movement exists; its number of Grundy is 0.

2. For a pair (a, b), build all the numbers that can be reached by removing 1, 2 or 3 A alone, B alone, or both simultaneously. The number of Grundy is the mex: the smallest non-negative integer absent from this set. Calculates by increasing amounts a+b, as all successors have a smaller sum.

3. Start of the table, useful for checking the convention: a\b | 0 1 2 3 4 5 6 7 ; 0 | 0 1 2 3 0 1 2 3 ; 1 | 1 2 0 4 1 2 0 4 ; 2 | 2 0 1 5 3 0 1 2 ; 3 | 3 4 5 6 2 7 4 5 ; 4 | 0 1 3 2 0 1 5 3 ; 5 | 1 2 0 7 1 2 0 4 ; 6 | 2 0 1 4 5 0 1 6 ; 7 | 3 4 2 5 3 4 6 5.

4. The values of the given pairs are: g(21,16)=1, g(18,23)=4, g(22,19)=3. Their XOR is 6. It is non-zero: the position is winning, provided you join a null XOR.

5. For each move in pair k, replace its old value g with the new value g′: new XOR = old XOR XOR g XOR g′. The winning moves are exactly: 1: B−1 → (21, 15); 2: A−2 → (16, 23); 2: A−3 and B−3 → (15, 20).

6. Check of all other moves: 1: A−1 → (20, 16) : XOR 7 ; 1: A−1 and B−1 → (20, 15) : XOR 5 ; 1: A−2 → (19, 16) : XOR 5 ; 1: B−2 → (21, 14) : XOR 7 ; 1: A−2 and B−2 → (19, 14) : XOR 4 ; 1: A−3 → (18, 16) : XOR 3 ; 1: B−3 → (21, 13) : XOR 5 ; 1: A−3 and B−3 → (18, 13) : XOR 7 ; 2: A−1 → (17, 23) : XOR 1 ; 2: B−1 → (18, 22) : XOR 3 ; 2: A−1 and B−1 → (17, 22) : XOR 2 ; 2: B−2 → (18, 21) : XOR 2 ; 2: A−2 and B−2 → (16, 21) : XOR 3 ; 2: A−3 → (15, 23) : XOR 7 ; 2: B−3 → (18, 20) : XOR 4 ; 3: A−1 → (21, 19) : XOR 2 ; 3: B−1 → (22, 18) : XOR 4 ; 3: A−1 and B−1 → (21, 18) : XOR 5 ; 3: A−2 → (20, 19) : XOR 7 ; 3: B−2 → (22, 17) : XOR 5 ; 3: A−2 and B−2 → (20, 17) : XOR 4 ; 3: A−3 → (19, 19) : XOR 1 ; 3: B−3 → (22, 16) : XOR 3 ; 3: A−3 and B−3 → (19, 16) : XOR 7. They each leave a non-zero sum: a winning answer exists for the opponent.

7. Why this works: when the XOR of the components is zero, a move changes one Grundy number to a different value, making the XOR nonzero. When the XOR is nonzero, choose a component with the highest set bit of the XOR. Its target g′ = g XOR XOR is less than g. By the definition of mex, every value below g is reachable. This lets you restore the XOR to zero after each opposing move.

8. The game ends because the total number of tokens decreases. Keeping the XOR at zero after each of your moves ensures you are never the player unable to move: (0,0) for every pair gives XOR zero. You therefore take the final token. This reasoning covers every opponent response.

9. For the verification of high values, apply exactly the recurrence, up to a maximum of 23 of this exercise. A symmetry g(a,b)=g(b,a) is allowed. Does not assume periodicity from the first eight lines: the evidence here uses the complete table.

Common pitfalls

- Apply the XOR of the heap sizes: it is the Grundy numbers of the PAIRS that combine.

- Forget simultaneous withdrawals, or allow two different quantities when withdrawing simultaneously.

- Transpose a poverty rule to this normal rule.

Verification

Accurate dynamic programming of Grundy numbers, control of the mex and all the first moves. The sum property is demonstrated in the fix.

Created by Tokiniaina Hervé

Difficulty levels reflect the techniques used in this collection.